



**Illustration of uncertainties
on the plots for Executive summary**

Yohei Tsukahara and Hiromu Fukuda

Highly Migratory Resources Division, Fisheries Resources Institute,
Japan Fisheries Research and Education Agency
2-12-4, Fukuura, Kanazawa-ku, Yokohama, Kanagawa 236-8648, JAPAN

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Background

The PBFWG has been used the bootstrap procedure to illustrate the uncertainty in the estimated demography (e.g. spawning stock biomass (SSB), recruitment, and Spawner Potential Ratio (SPR)) for the assessment of the stock status. Also, it was used to carry its uncertainty in the age structure at the assessment terminal year and the historical recruitments, which were used for their future projection (ISC 2022).

However, in the 2024 stock assessment, the PBFWG recognized that the maximum likelihood estimates (MLEs) of the terminal SPR was below 90% confidence intervals derived from bootstrap replicates. The WG decided to use normal approximation of Hessian matrix for the description of uncertainty of the results of the base-case model to assess the status of the stock.

This document presents how to illustrate the uncertainties on the result plots based on the one derived from Hessian matrix. The confidence intervals for the trajectories were calculated by standard deviation multiplied by certain values. For the uncertainty in a phase of plot for stock status, I tried to apply the multi-variate log-normal distribution. This document has been revised after PBFWG meeting according to the discussion and decision there for the sake of memorandum record for next assessment.

Trajectories of Total Biomass, SSB and Recruitment

Using Hessian matrix with normal approximation makes multivariate optimization easy and quick. The positively defined Hessian matrix indicates that every parameter finds a solution as convex function, then the standard deviation (SD) can be available for the estimated parameters and some derived quantities such as annual SSB and recruitment in SS3 report file without any setting changes from base case model. On the other hand, for the SD for total biomass, the authors had estimated CV for the total biomass, assuming that the CVs of recruitment propagate in the cohort for immature fish and the CVs of SSB is representative of error for matured fish, i.e. age-3 and older. However, during the meeting, it was pointed out that the SD for total biomass also can be estimated by original function of the SS software by specifying the output format in control file. We therefore revised the

way to calculate the CV in accordance with the suggestion from the WG.

SS3 has a function to calculate the summary biomass (SmryBio) and its SD, which are aggregated biomass for older than a specified age. This is the function just for the output purpose, and hence there is no impact on the estimation of stock dynamics. The minimum age for summary biomass customary set to age 1 in the previous PBF assessments although the summary biomass output was never used for any purpose. The authors have changed that to age 0 to calculate the summary biomass as total biomass from age 0 to age 20. Although the annual summary biomass was recorded in the “ss_summary.sso”, there is a need to change a few settings in the very end part of control file to calculate its SD. There is a section to specify the requirement of additional reporting, e.g. for selectivity and summary biomass and so on, although it was set to 0 as no additional SD reporting. We have changed it to 2 to add all options for whether or not we need selectivity, size, numbers, M, dynamic Bzero and summary biomass. And then we made a setting to output additional SD only for the total biomass. The changes were as follows;

(Starter file)

Before:

1 # min age for calc of summary biomass

After:

0 # min age for calc of summary biomass

(Control file)

Before:

0 # (0/1/2) read specs for more stddev reporting ...

After:

2 # (0/1/2) read specs for more stddev reporting ...

0 0 0 0 # Selectivity: ...

0 0 # Growth: ...

0 0 0 # Numbers-at-age: ...

0 0 # Mortality: ...

0 # Dyn_Bzero: 0 to skip, 1 to do, 2 w/ recr

1 # SmryBio: 0 to skip, 1 to do

With these changes, the SD for total biomass was successfully obtained in the SS calculation. The decision point here is a range of confidence intervals. The 90% and 95% confidence intervals were estimated by multiplier 1.64 and 1.96, respectively. The Figure 4 showed that the confidence intervals were asymmetric for upward and downward naturally, whereas the confidence intervals derived from bootstrap replicates were somewhat biased. Also, the result may indicate that the impression and information based on each confidence interval were not much different.

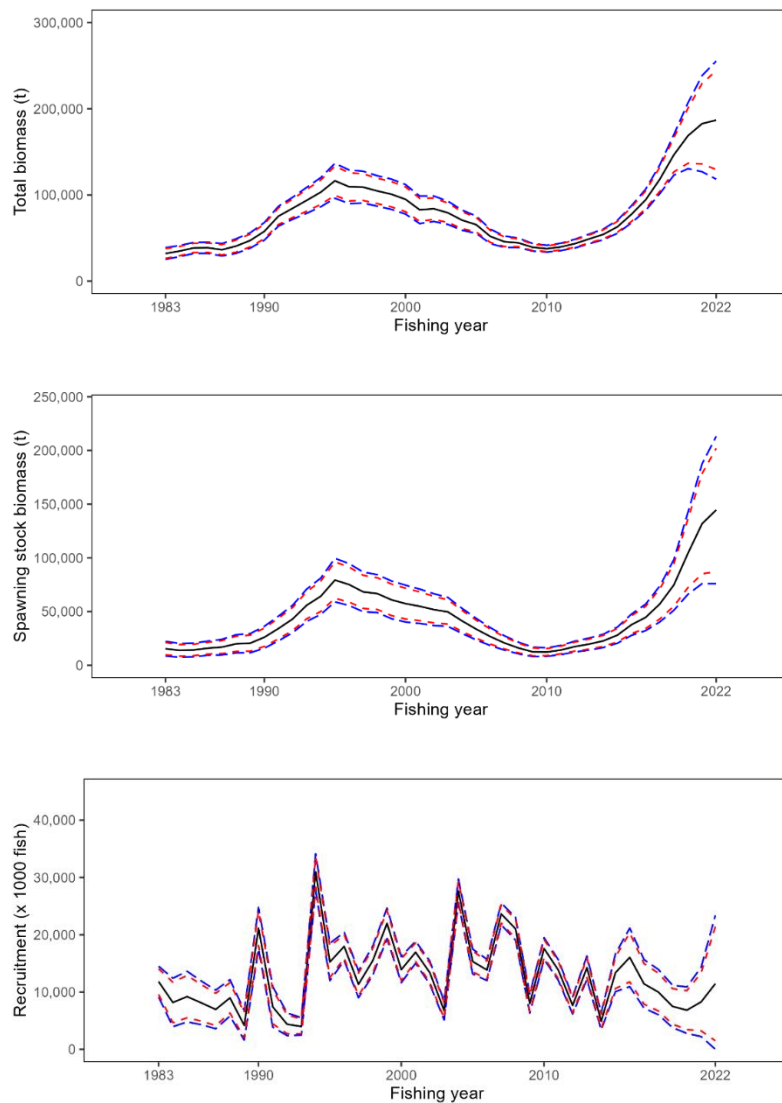


Figure 3. Trajectories of Total biomass (Top), SSB (Middle) and Recruitment with 90% (Red) and 95% (Blue) intervals.

Multi-variate log-normal for the uncertainty in a plot for stock status.

The PBFWG has been used the relative SSB to 20%SSB|F=0 for x-axis and spawning potential ratio (SPR) for y-axis to illustrate the phase of stock status, hereafter called phase plot, instead of general Kobe plot. As in the previous section, the biased recruitment estimates in the recent couple of years in bootstrap replicates made the SPR values biased as well. As a result, the 300 termina points in 2022 FY from bootstrap replicates was not overlapped the MLE in 2022 FY. The PBFWG discussed the usefulness of the multi-variate log-normal (MVLN) method to illustrate the uncertainty in the plot. The calculation and data generation were conducted by “rmvnorm” function in “mvtnorm” library and R version 4.3.2.

The author mimicked a method of the MVLN used in the ICCAT (Kell et al. 2023). Firstly, the values of “Fraction of Depletion denominator” was changed from 1 in the base case calculation to 0.2 for the MVLN to be relative SSB to 20%SSB|F=0. Then, annual correlations between Bratio and raw SPR values were extracted. The Bratio and SPR estimation had moderate correlation relationship for the assessment period (Figure5), which was recorded in the “covar.sso”. The average and median of the historical correlation were -0.406 and -0.407, respectively.

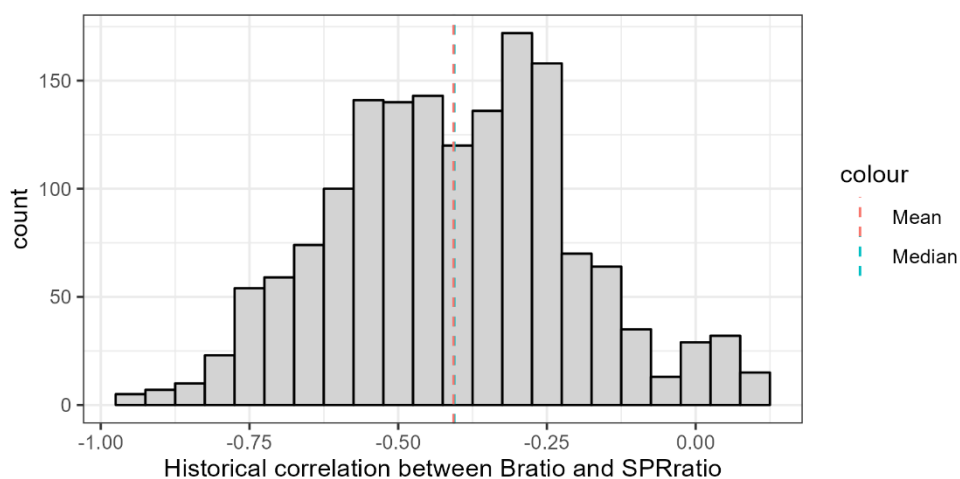


Figure 5. Annual correlation values between Bratio and SPR during assessment period.

This document used the median of historical correlation for calculation of covariance value. The expected values for MVLN were just log-transformed value of SPR and Bratio at the terminal year. The variance (Var) for each variable was calculated as follows;

$$Var = \log\left(1 + \left(\frac{\text{standard error}}{\text{estimated values}}\right)^2\right).$$

The covariance (Cov) between Bratio and SPR was estimated as follows;

$$Cov = \log\left(1 + \text{correlation}_{\text{median}} * \sqrt{\text{var}_{\text{SPR}} * \text{var}_{\text{Bratio}}}\right).$$

Finally, variance-covariance matrix (VCM) was defined as follows;

$$VCM = \begin{bmatrix} Var_{SPR} & Cov_{Bratio,SPR} \\ Cov_{Bratio,SPR} & Var_{Bratio} \end{bmatrix}$$

The author generates 300 random numbers in accordance with the MVLN distribution to see if there are differences by choices of random seed. One of the contention points for MVLN procedure is regarding the random seed, because the MVLN is just random number generation. Figure 6 indicated that there were little differences in general trend by random seed, randomly generated uncertainties had seemingly negative correlation as it was defined by VCM. Therefore, Random seed 1 was used for the following plots. For the plots in the Executive summary and assessment report, the number of generated points will be 1 million to sufficiently reduce the impact of choice of random seed.

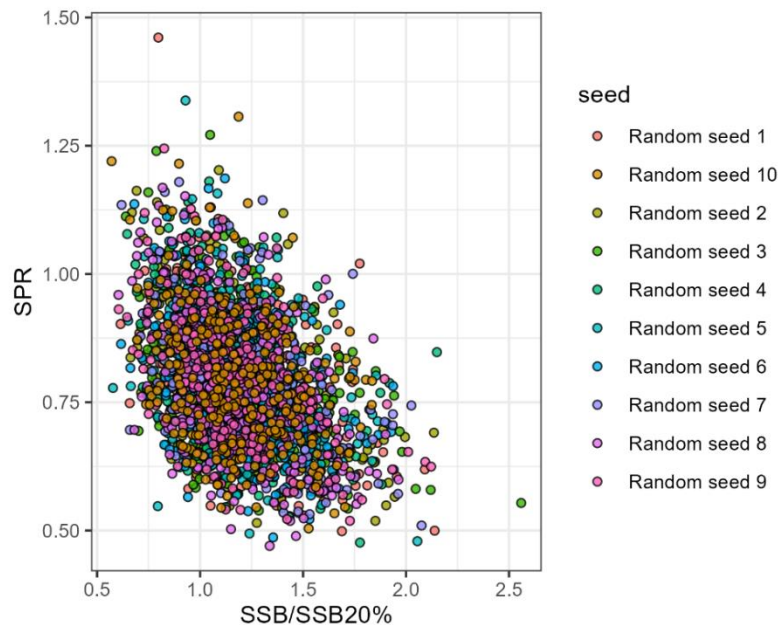


Figure 6. Relationship between the relative SSB over the 20%SSB0 and SPR (1-SPR) for the generated terminal year stock status from MVLN by different random seed.

It should be noted that several points in Fig. 6 showed that SPR (1-SPR) was higher than 1.00. Whereas such value is theoretically impossible in the SPR calculation (i.e. harvesting more than existing), the MVLN can generate those points just from the correlation among the variables without calculating the population dynamics. Those values will not be presented in the phase plot, but it should be considered as a part of uncertainty. To illustrate the uncertainty in phase plot at the terminal year, the contour plot with the circular lines was suggested (Fig. 7). The probability from 60% to 90% with 10% interval was depicted in contour plot, using the “geom_hdr_lines” function in “ggdensity” library. Due to the existence of the points higher than 1.00 SPR (1-SPR), the circular line for 80% and 90% probabilities are not completely closed circles. This plot would be included in the Executive summary and assessment report.

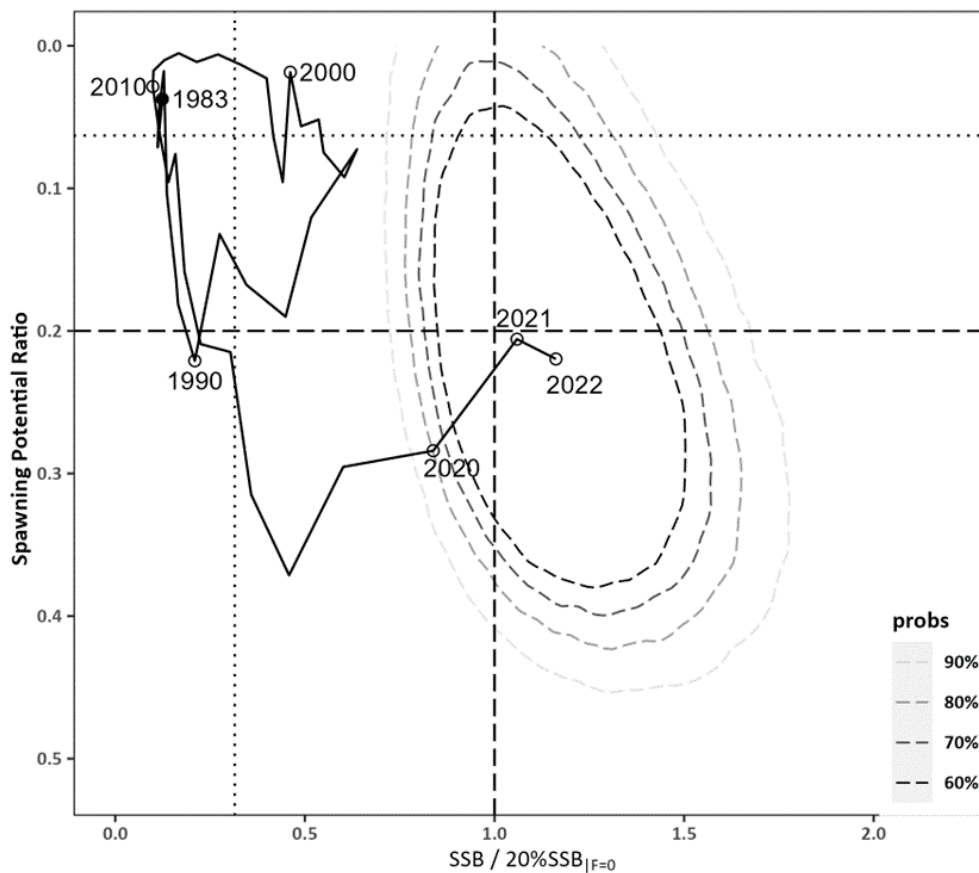


Figure 7. Suggested phase plot with uncertainties illustrated by contour plot.

References

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Kell, L., Rice, J., Coutney, D. and Winker, H. (2023) APPROXIMATION OF KOBE POSTERIORES FROM STOCK SYNTHESIS FOR NORTH ATLANTIC BLUE SHARK. Collect. Vol. Sci. Pap. ICCAT, 80(4) 837-858.