

ISC/12/PBFWG-3/01

Estimate the frequency distribution of steepness for PBF

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Nov. 2012

Working document submitted to the ISC Pacific bluefin tuna Working Group, International Scientific Committee for Tuna and Tuna-Like Species in the North Pacific Ocean (ISC), 10-17 November 2012, Honolulu, Hawaii. **Document not to be cited without author's permission.**

Introducti

Abstract

The concept and the value of steepness is one of the important parameters in the stock assessment, since steepness affects stock-recruitment dynamics and gives benchmark of fish stock management. In this document, we estimated the frequency distribution of steepness for Pacific Bluefin Tuna (PBF) by using the similar Monte Carlo simulation procedures used by Mangel et al. (2010). The estimated frequency distribution of steepness for PBF indicates that the stock recruitment relationship is sparse (i.e. high steepness value). The probability mass of the steepness values is concentrated from 0.997 to 0.999. This finding justified the working group's decision to use 0.999 as of the steepness value at up-coming stock assessment with 0.8 and 1.0 for sensitivity analysis based on estimation in this document and Mangel's result.

Introduction

The spawner-recruitment relationship, i.e. relationship between spawning stock biomass (SSB) and recruitment (*R*), is an important issue in stock assessments, since recruitment is one of the important indices to evaluate stock status as well as a benchmark for fish management. One important parameter which describes the Stock Recruitment relationship is steepness (see definition in the next section). In the stock assessment meeting in February, 2012, Iwata et al. (2012) estimated the steepness of Pacific Bluefin Tuna (PBF) by using the deterministic version of the method of Mangel et al. (2010),obtaining a point estimate of steepness value. In this document, the frequency distribution of steepness for PBF was estimated by using the similar calculation rules used by Mangel et al (2010).

Materials and methods

Concept of steepness

The definition of steepness, as the fraction of recruitment from an unfished population (R_0) to the recruitment level when the spawning stock biomass is reduced to 20% of its unfished level (B_0), is firstly proposed by Goodyear (1977, 1980). After that, Mace and Doonan (1988) assumed that reproduction follows a type of Beverton-Holt stock recruitment relationship so that when spawning biomass is B, the resulting recruitment R(B) is,

$$R(B) = \frac{B}{\alpha + \beta B}$$

where α and β are parameters. Let us denotes steepness as *h*, then following relationships are satisfied;

$$R_0 = \frac{B_0}{\alpha + \beta B_0}$$
, $hR_0 = \frac{0.2B_0}{\alpha + 0.2\beta B_0}$

where B_0 represents biomass without fishing and let us define $R(B_0)_0$ as R_0 . α and β are determined by the followings;

$$\alpha = \frac{B_0}{R_0} \frac{1-h}{R_0 4h}, \ \beta = \frac{5h-1}{4hR_0}.$$

In general, a relatively high value of steepness means that the stock recruitment relationship is sparse (low); stock recruitment curve being close to parallel to the X-axis.

Model to derive steepness

In this document, the steepness of PBF is calculated deterministically by modified methods of Mangel et al., (2010) together with other biological information. Firstly, r denotes the fraction of female at birth. M denotes the rate of natural mortality and α_p denotes the units of new biomass per existing spawning (female) biomass per time period. The steepness can be estimated by non-age-structured model as follows:

$$h = \frac{\frac{\alpha_p(1-r)}{M}}{4 + \frac{\alpha_p(1-r)}{M}},$$
 (1)

The equation (1) without sex ratio, was derived from Myears et al. (1999). This particular form is proposed by Mangel et al. (2010). Steepness of age structured model is as follows;

$$h = \frac{\alpha_{s}(1-r)\frac{W_{f,m}}{M}}{4 + \alpha_{s}(1-r)\frac{W_{f,m}}{M}},$$
(2)

 $\overline{W}_{f,m}$ is the average biomass of a spawning female and α_s denotes the expected number of new individuals per unit spawning biomass. Let us consider, α_p , α_s , and $\overline{W}_{f,m}$. In the equations (1) and (2), relationship among these parameters, i.e. $\alpha_s \approx \alpha_p / \overline{W}_{f,m}$, is satisfied implicitly. Therefore, once α_p is calculated, remainders are trivial.

Details of calculation

Using Mangel et al. (2010), let us consider two periods; early life period and grown-up periods. Various biological characteristics and parameters, in particular natural mortality, seem to be quite different in PBF between these two periods. Fish in the second period, grown-up period, follows most of the biological parameters estimated to the adult fish. However fish in the early life period are considered to have very distinct parameters. This period (early life period) represents time duration, in days, i.e. d_m days after hatch: d_m is defined by Mangel by using von Bertalanffy parameter (theoretical age at which size is zero: a_0) as, $-365 \times a_0$. Second period (adult period) is after d_m days. To find α_p , let us assume a female fish of age *a*, with mass during adult period w(a) (W(0) means the weight at start of adult period), length during adult period L(a), probability of being mature $p_m(a)$ by age. $F_B(w)$ is defined as batch fecundity of this individual in a single spawning event and s_t represents spawning frequency. Each egg is expected to contribute $W(0) \times S_t(d_m)$ to new biomass. Then α_p can be derived as follows;

$$\alpha_p = \frac{F_B(a) \times W(0) \times S_l(d_m) \times s_f}{W(a)}$$
(3)

W(0) is interpreted as $\overline{W}_{f,m}$ (Mangel et al., 2010).

Next let us see the details of dynamics during early life period and adult. We calculate the temporal changes of survival rate by using dry weight of egg – natural mortality relationship (McGurk, 1986). Let us define wet mass $w_l(d)$ of a larva at an age (in days) and assume exponential growth during early life period,

$$w_l(d) = w_l(0) \times \exp(k_l d),$$

where k_l is determined by $k_l = \log(w_l(d)/W(0))/d_m$, since it is defined as the individual growth from initial weight of early life period $w_l(0)$ to initial weight of adult period W(0) during d_m days of early life period.

To compute an expected mass that an egg contributes to future biomass, the survival of individuals during early life period has to be decided, before being entered to the population dynamics model. The conversion factor from dry weight, w_{dry} , to wet weight is

$$w_{i} = 4.76 \times w_{dry}$$
 (Kamler, 1994).

 $M_l(w_{dry})$ is defined as a daily mortality rate of an individual of dry mass W_{dry} . McGurk (1986, Equation 7) concluded empirically that if $M_l(w_{dry})$ denotes a daily mortality rate of an individual of dry mass w_d . If w_{dry} is less than 0.00504(g), then

$$M_l(w_{dry}) = 2.2 \times 10^{-4} \times w_{dry}^{-0.85}$$

or by McGurk (1986, Equation 2),

$$M_l(w_{dry}) = 5.26 \times 10^{-3} \times w_{dry}^{-0.25}.$$

Assuming survival at day d_m with probability $S_l(d)$ (with $S_l(0) = 1$); natural mortality at age a, $M_a(d)$ (with $M_a(0) = 0$) and accumulated mortality are

$$S_l(d) = \exp(-\sum_{i=0}^d M_l(w_{dry}(w_l(j)))), \ M_a(d) = \sum_{i=0}^d M_l(w_{dry}(w_l(j))).$$
(4)

In the adult period, the weight and fork length relationship is considered as follows;

$$W(a) = 0.001 \cdot L(a)^{2.4}$$
, (Mangel et al., 2010)
 $L(a) = L_{\infty} \{ 1 - e^{-k(a-a_0)} \}.$

Individuals grow up following the above equation until they reach maximum age (a_{max}) . Finally, relationship between batch fecundity (F_B ; million eggs) and fresh gonad mass (M_G ;g) is

$$F_B = 3.24 \times 10^5 \times L(a) - 5.21 \times 10^7$$
 (Chen et al. 2006).

The probability that a fish is mature at age *a* is,

 $p_m(a) = e^{(a-a_{50})/\text{slope}} / \{1 + e^{(a-a_{50})/\text{slope}}\},\$

where a_{50} is the age at which 50% of a cohort is mature (see Table 1). Usually, a *slope* is needed in the description of $p_m(a)$. However whether or not the steepness value critically changes is depend on the value of *slope*. So we set the value of slope as 1. The age of 50% maturity, a_{50} is considered to be 5 years old in this study, since estimated length-at-50% maturity roughly corresponds to age 5 (Anon. 2008).

In this document, based on equation (3) and (4), we calculate an approximate value, $\tilde{\alpha}_p$, as follows,

$$\widetilde{\alpha}_p = \frac{\sum_{a} (F_B(a) p_m(a)) \times W(0) \times S_l(d_m) \times s_f}{\sum_{a} W(a)}$$

On the other hand, an approximate value, $\tilde{\alpha}_s$ can be derived from the following formulation, $\tilde{\alpha}_s \approx \tilde{\alpha}_p / \overline{W}_{f,m}$. So, the definition of $\overline{W}_{f,m}$ is introduced as,

$$\overline{W}_{f,m} \approx \frac{\left(\sum_{age=1}^{a_{max}} S(a) W_f(a) p_{f,m}(a)\right) \left(1-e^{-M}\right)}{\left(1-e^{-Ma_{max}}\right)}$$

where $W_f(a)$ and S(a) are weights of female at age *a* and the survival probability at age *a*.

Simulation procedures

To estimate the frequency distribution of steepness values, Monte Carlo simulation was applied. In the simulation, we estimate frequency distribution of steepness for 200 groups, and each group consists with 200 individuals. After that, the confidence interval for each steepness value would be analyzed.

For calculation of steepness, we should set lifetime (maximum age) and mortality rate until death for each individual. Let us explain how to determine the maximum age and mortality rate. For simplicity, the individual will die in the simulation if the individuals reach at age, 20. That is, individuals do not survive beyond age 20 in these simulations.

Firstly, the natural mortality rate (m) for each age class set as the random value generated

following gamma frequency distribution, $f(m) = \frac{\lambda^{\nu}}{\Gamma(\nu)} e^{-\lambda m} m^{\nu-1} dm$ where m is natural

mortality (per year), and ν and λ are constant parameters. In this formulation the expected value of *M* is $E(M) = \overline{M} = \nu / \lambda$ and the coefficient of variance is $CV(M) = 1 / \sqrt{\nu}$. Following the generated natural mortality, survival rate for each age class is determined.

Secondly, we generate the random value following uniform distribution. If the value is higher than the survival rate at some age, then that age is the terminal age of that individual. On the contrary, the individual still survive if the value is lower than the survival rate.

Finally, by using the estimated maximum age, we estimate steepness after calculating $\tilde{\alpha}_p$ and $\tilde{\alpha}_s$.

So we have 200 frequency distributions for 200 groups and the confidence intervals were calculated.

Results and Discussions

The parameters used in this analysis are shown in Tables 1, 2 and 3, and the early life period is defined as $d_m = -365 \times a_0$ (same definition as lwata et al., 2012 and Mangel et al., 2010). Natural mortality for age 0 (i.e. 1.6) applied to stock assessment model is calculated from the individuals (of over 20cm ub fork length), maybe 60 or 60 some days after hatched). This means that the early life period calculated by using Bayliff's growth curve (i.e. 257 days) is too long and growth curves except Baylif's one in table3 have similar ambiguity (see Table. 3). The definition ($d_m = -365 \times a_0$) is still much ambiguous as given in the previous work (lwata et. al., 2012).

The same calculation method as lwata et al., 2012 is usede in this document except the setting of individuals of maximum age. In the lwata et al., (2012) we assumed that any individuals can survive until maximum age, i.e. age 20. However in this document, each individual has different life span with the maximum limit of age 20.. For the calculation of maximum age, we have to know about the parameters, i.e. average natural mortality \overline{M} , and constant values v and λ . The average natural mortality \overline{M} for several scenarios are given in Table 2. So the frequency distribution for generating natural mortality can be completely determined if the value v is given. However, we have no available information about the value v except for Bayliff's growth curve (Mangel et al., 2010 stated that v is 9.7 for Bayliff's case). Therefore, we do sensitivity analysis also for v=7 and 11 in addition to 9.7. If v is high, the frequency of individuals with high mortality is expected higher than in the case of low v (see Fig. 1). Therefore, the lifetime will be expected shorter than the low v.

For applying several scenarios for natural mortality and growth curves on different models (production model or age structured model), no big significant differences appear in estimated frequency distribution of steepness (see. Fig. 3-5). That is, estimated steepness values are all located near the upper limit, one (see. Fig 3 shows frequency distribution for $\nu = 9.7$. Fig 4 shows cumulative frequency distribution for $\nu = 9.7$). The stock recruitment relationship is very sparse.

As a result, the estimated values in frequency distribution of steepness are near to one, i.e. within the interval of 0.997-0.999 for both cases of production model and age structured model. So it is appropriate that the steepness value at the coming stock assessment be set as 0.999. However, revised results given in the erratum of Mangel et al. (2010) indicated possible range of steepness of 0.8-1.0. The difference between their result and our result

may come from the derivation of $\tilde{\alpha}_s$ and $\tilde{\alpha}_p$. Therefore, at the coming stock assessment, the value in the range of 0.8 to 1.0 is recommended for the sensitivity analysis from our results and Mangel's results.

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Table 1. The parameters used in this docume	ent.
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Symbol	Meaning	PBFT
R	Sex ratio	0.5
Am	Maximum age in simulation	12
Sf	Spawning frequency	Every 3.3 days during 6 weeks = 12.73
8 50	Age at which 50% of a cohort is mature	5

Table 2. Natural mortality scenarios

Symbol	meaning	M scenario options
М	Average natural mortality	ISC 2008 {2.33+0.2(1+am)}/(1+am)
		(M0=1.6, M1=0.46, M2=0.27, M3+=0.2)
		ISC 2010 {1.986+0.25(1+am)}/(1+am)
		(M0=1.6, M1=0.386, M2+=0.25)

Table 3. Growth parameter options

	Asymptotic size, L_{∞}	Growth rate, <i>K</i>	Age at which size is 0, <i>a</i> ₀	Early life history
Bayliff, 1994	320.5	0.1035	-0.7034	256
Shimose et al. (2009)	249.6	0.173	-0.254	92
Shimose et al. (2011) Both sexes	252.1	0.165	-0.259	94
Shimose et al. (2011) female only	240.5	0.192	-0.089	32
Stock Synthesis (May-June, 2012)	254.41	0.15743	-0.560695	204



Figure 1. The gamma frequency distribution $(f(m) = \frac{\lambda^{\nu}}{r(\nu)}e^{-\lambda m}m^{\nu-1}dm)$ for natural mortality of PBF. In the case, we use average, \overline{M} as 0.276. The mean of gamma frequency distribution is $E(M) = \overline{M} = \nu/\lambda$ and the coefficient of variance is $CV(M) = 1/\sqrt{\nu}$.



Figure 2. Frequency distribution of steepness value for several assumptions ($\nu = 9.7$). The difference between a-1 and a-2 (b-1 and b-2) is only the range of x axes (0.2-1.0 or 0.997-1.0). In first (second) column of each figure, production (age structured) model was used for estimation. In first to fifth row, growth curve in Bayliff (1994), Shimose et al. (2009), for both sex in Shimose and Takeuchi (2012), , for female only in Shimose and Takeuchi (2012), used in first PBF stock assessment WG 2012.



Figure 3. Cumulative frequency distribution of steepness value for several assumptions ($\nu = 9.7$). The difference between a-1 and a-2 (b-1 and b-2) is only the range of x axes (0.2-1.0 or 0.997-1.0). In first (second) column of each figure, production (age structured) model was used for estimation. In first to fifth row, growth curve in Bayliff (1994), Shimose et al. (2009), for both sex in Shimose and Takeuchi (2012), , for female only in Shimose and Takeuchi (2012), used in first PBF stock assessment WG 2012.



Figure 4. Frequency distribution of steepness value for several assumptions (ν =7). The difference between a-1 and a-2 (b-1 and b-2) is only the range of x axes (0.2-1.0 or 0.997-1.0). In first (second) column of each figure, production (age structured) model was used for estimation. In first to fifth row, growth curve in Bayliff (1994), Shimose et al. (2009), for both sex in Shimose and Takeuchi (2012), , for female only in Shimose and Takeuchi (2012), used in first PBF stock assessment WG 2012.



Figure 5. Frequency distribution of steepness value for several assumptions ($\nu = 11$). The difference between a-1 and a-2 (b-1 and b-2) is only the range of x axes (0.2-1.0 or 0.997-1.0). In first (second) column of each figure, production (age structured) model was used for estimation. In first to fifth row, growth curve in Bayliff (1994), Shimose et al. (2009), for both sex in Shimose and Takeuchi (2012), , for female only in Shimose and Takeuchi (2012), used in first PBF stock assessment WG 2012.