#### **Age-Based Analyses of Potential Biological Reference Points**

### for the Western and Central North Pacific Swordfish (Xiphias gladius) Stock

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### Abstract

Age-based demographic analyses were used to determine a suite of candidate biological reference points for the Western and Central North Pacific swordfish stock for consideration by the ISC Billfish Working Group. Life history data and results from the recent age-structured stock assessment modeling of this stock were used to compute the fishing mortality reference points  $F_{MSY}$ ,  $F_{MAX}$ ,  $F_{0.1}$ ,  $F_{MED}$ , and  $F_{SPR}$ . The same information was used to compute the biomass reference points  $B_{MSY}$ ,  $B_{MAX}$ ,  $B_{0.1}$ ,  $B_{MED}$ , and  $B_{SPR}$ . The percentage of maximum yield and spawning biomass per recruit were summarized to compare the relative yield and stock conservation benefits of the various fishing mortality reference points. Similarly, the ratios of reference biomass, recruitment, and yield to the values at MSY were also summarized to compare the relative stock conservation and yield benefits of the various biomass reference points.

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## Introduction

This Working Paper summarized analyses of potential biological reference points for the Western and Central North Pacific (WCPO) swordfish stock under consideration by the ISC Billfish Working Group. This work focused on age-based reference points derived from life history information and results from the age-structured assessment model for WCPO swordfish reported by Courtney and Piner (2010).

### Methods

Swordfish life history and fishery information were collected from the combined-sex model in Courtney and Piner (2010). Natural mortality, fraction mature, length and weight at age were compiled from the original life history data input to the Stock Synthesis model for the WCPO stock.

The aggregate fishery selectivity for the age-based reference point calculations was derived from the catch-weighted estimates of quarterly fishery selectivity by fishing fleet during 1994-2006 as estimated in the Stock Synthesis model. The quarterly fishery selectivity at age vector ( $\underline{S}_{Y,Q}$ ) was estimated for each year from 1994 to 2006 as the ending year (2006) selectivity at age by quarter and fleet vector ( $\underline{S}_{2006,Q,f}$ ) weighted by the percent catch in numbers vector ( $P_{Y,Q,f}$ ) by year and quarter and fleet, i.e.,  $\underline{S}_{Y,Q} = \sum_{f} P_{Y,Q,f} \cdot \underline{S}_{2006,Q,f}$ . The aggregate quarterly fishery selectivity at age vector ( $\underline{S}_{Q}$ ) was estimated as the average quarterly fishery selectivity for the years 1994 to 2006, i.e.,  $\underline{S}_{Q} = \frac{1}{N_{Y}} \sum_{Y} \underline{S}_{Y,Q}$ . Estimates of fishery selectivity at length and age were available for nine fleets (Courtney and Piner 2010). Estimated fishery selectivity at length and age were constant for most fleets during the years 1994 to 2006 (Courtney and Piner 2010). As a simplification, ending year (2006) selectivity at age by quarter and fleet for all years during 1994 to 2006 ( $\underline{S}_{Y,Q,f}$ ).

Swordfish cohorts were assumed to spawn in quarter 2 (April-June) of each year and age-0 juveniles were assumed to recruit in quarter 3 (July-September) of each year. Computations of spawning biomass and yield per recruit are described below.

Spawning biomass per recruit was calculated as the sum of expected spawning biomass at age per recruit as a function of survival at age over a cohort's life span using the NOAA Fisheries Toolbox YPR program (http://nft.nefsc.noaa.gov/YPR.html). In this calculation, spawning biomass per recruit (SPR) was a function of fishing mortality (F) at equilibrium (Figure 1). For each F, the value of SPR was the expected lifetime spawning biomass (as a proxy for egg production) per recruit at that fishing mortality (F) with fishery selectivity at age vector (P) and was determined by life history and fishery parameters (Table 1). The necessary life history parameters included the probability of maturity at age vector (D), the spawning weight (or fecundity) at age vector  $(W_S)$ , and the natural mortality at age vector (M) where no stockrecruitment relationship was assumed (Gabriel et al. 1989). Because some fisheries can produce high fishing mortalities prior to the spawning season within a calendar year, the fractions of natural  $(C_M)$  and fishing mortality  $(C_F)$  expected to occur prior to the spawning season are additional parameters (see Gabriel et al. 1989); these fractions were set to zero for swordfish because mortality prior to spawning in quarter 2 was assumed to be negligible. Given an agespecific natural mortality rate M<sub>a</sub> for each age class, the expected lifetime spawning biomass per recruit at fishing mortality F is the sum of expected spawning biomass at age times the probability of survival to age over the cohort's lifespan which includes up to As years of survival in the plus group where  $Z_A = M_A + P_A F$ .

(1.1)  

$$SPR(F) = \sum_{a=1}^{A-1} D_a W_{s,a} \cdot \exp\left(-C_M M - C_F P_a F - \sum_{k=1}^{A-1} Z_k\right) + D_A W_{s,A} \cdot \exp\left(-C_M M - C_F P_A F - \sum_{k=1}^{A-1} Z_k\right) \cdot \frac{1 - \exp\left(-Z_A A_s\right)}{1 - \exp\left(-Z_A\right)}$$

The values of fishing mortality ( $F_{SPR}$ ) that would produce fixed percentages of the unfished spawning biomass per recruit ranging from 10% to 90% by 10% were calculated as examples of potential biological reference points for WCPO swordfish.

Yield per recruit (YPR) was calculated as the expected yield in biomass per recruit from harvesting a cohort over its life span. This function of fishing mortality at equilibrium was computed using the YPR program (http://nft.nefsc.noaa.gov/YPR.html). The value of YPR was the expected contribution of fishery yield per recruit at a fishing mortality F (Figure 2) with fishery selectivity at age vector (P) and was determined by life history and fishery parameters (see Table 1). In this case, the necessary input parameters were the same as used for SPR with the exception of the catch weight at age vector ( $\underline{W}_C$ ). Given an age-specific natural mortality F was the sum of expected yield at age times the probability of survival to age with age-specific natural mortality over the cohort's lifespan.

(1.2) 
$$YPR(F) = \sum_{a=1}^{A-1} \frac{W_{C,a} P_a F}{Z_a} (1 - \exp(-Z_a)) \cdot \exp\left(-\sum_{k=1}^{a-1} Z_k\right) + \frac{W_{C,A} P_A F}{Z_A} \exp\left(-\sum_{k=1}^{A-1} Z_k\right)$$

The fishing mortality that produced that maximum YPR value ( $F_{MAX}$ ) was computed as a potential biological reference point. Similarly, the fishing mortality that produced 10% of the maximal increase in YPR ( $F_{0.1}$ ) was also another potential reference point that was independent of stock-recruitment dynamics.

Biological reference points associated with the maximum sustainable yield (MSY) level for the WCPO swordfish were constructed from life history information and the age-structured assessment outputs. The age-structured stock assessment of WCPO swordfish in Stock Synthesis used a Beverton-Holt stock-recruitment curve formulated in terms of steepness (*h*)

(1.3) 
$$R(B) = \frac{4hR_0B}{B_0(1-h) + B(5h-1)}$$

where R was the number of recruits (thousands), B was spawning biomass (mt), h was steepness fixed at 0.9 based on Myers et al. (1999) and Courtney and Piner (2010), unfished recruitment ( $R_0$ ) was estimated to be 691.4 (thousands), and unfished equilibrium spawning biomass ( $B_0$ ) was estimated to be 41,487.4 mt.

This stock-recruitment curve was used to compute MSY-based reference points using the NOAA Fisheries Toolbox YPR module. In particular, the Beverton-Holt curve relating recruitment (R, thousands of age-0 fish) to spawning biomass (B, mt) in the Fisheries Toolbox YPR module was

(1.4) 
$$R(B) = \frac{\alpha \cdot B}{\beta + B}$$

and the values of  $\alpha$  (7111.113) and  $\beta$  (1185.354) for the WCPO were found analytically from Stock Synthesis output as

(1.5) 
$$\alpha = \frac{4hR_0}{5h-1} \text{ and } \beta = \frac{B_0(1-h)}{5h-1}$$

The equilibrium spawning biomass B at fishing mortality F based on the Beverton-Holt curve was

(1.6) 
$$B(F) = \alpha \cdot SPR(F) - \beta$$

and the equilibrium recruitment at fishing mortality F was

(1.7) 
$$R(F) = \alpha - \frac{\beta}{SPR(F)}$$

Last, the equilibrium yield Y(F) at fishing mortality F was the product of equilibrium yield per recruit and recruitment at F (see, for example, Sissenwine and Shepherd (1987)) via

(1.8) 
$$Y(F) = YPR(F) \cdot \left(\alpha - \frac{\beta}{SPR(F)}\right)$$

The fishing mortality that produced the maximum sustainable yield ( $F_{MSY}$ ) was determined using a grid search over all possible values of F. This computation provided estimates of the MSYbased reference points  $F_{MSY}$  and the spawning biomass to produce MSY ( $B_{MSY}$ ).

The biological reference point  $F_{MAX}$ , defined as the fishing mortality that produced the maximum yield per recruit (Beverton and Holt 1957), was computed from the equilibrium yield-per-recruit curve using the YPR program. Similarly, the biological reference point F0.1, defined as the

fishing mortality where the slope of the YPR curve is one-tenth of the slope at the origin (Gulland and Boerema 1971), was computed using the YPR program.

The biological reference point  $F_{MED}$ , defined as the fishing mortality corresponding to the median of the observed distribution of recruits per spawning biomass ratios (see, for example, Mace and Sissenwine 1993), was computed using the age-structured assessment results of Courtney and Piner (2010). In particular, the estimated recruits per spawning biomass distribution from 1969-2004 (Figure 1) was used to determine the median value of R/B (Figure 2) and the median was used to compute the SPR and associated value of  $F_{MED}$  (Figure 3).

The biological reference point  $F_{CRASH}$ , defined as the fishing mortality associated with the value of SPR as the number of spawners approaches zero (SPR<sub>CRASH</sub>), was computed from the unfished spawning biomass per recruit ( $\Phi_0$ ) and the Beverton-Holt stock-recruitment parameters ( $\alpha$  and  $\beta$  estimated in units of spawning biomass). In particular, the value of SPR<sub>CRASH</sub> was equal to the inverse of the maximum lifetime reproductive rate (see Brooks et al. (2010)) or SPR<sub>CRASH</sub> =  $\beta/(\alpha \Phi_0)$  and the associated value of F<sub>CRASH</sub> was determined from the equilibrium SPR curve.

The values of the spawning biomass-based biological reference points  $B_{MSY}$ ,  $B_{MAX}$ ,  $B_{0.1}$ ,  $B_{MED}$ , and  $B_{SPR}$  were determined from the associated fishing mortality-based reference points and the equilibrium spawning biomass curve as a function of F.

The percentage of maximum yield and spawning biomass per recruit were summarized to compare the relative yield and stock conservation benefits of the various fishing mortality reference points. Similarly, the ratios of reference biomass, recruitment, and yield to the values at MSY were also summarized to compare the relative stock conservation and yield benefits of the various biomass reference points.

### Results

Results of the  $F_{SPR}$  calculations indicated that  $F_{SPR}$  ranged from F10%=0.48 qtr<sup>-1</sup> to F90% =0.01 qtr<sup>-1</sup> with ratios of reference F to adult natural mortality ranging from 5.5 to 0.1 (Table 2 and Figure 4). Maximum YPR was realized near F20% and the ratio of reference F to adult M was approximately 1 near F40%.

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The other fishing mortality reference points fell within the range of  $F_{SPR}$  values examined (Tables 2 and 3 and Figure 5), with the exception of  $F_{CRASH}$ . The value of  $F_{MSY}$  was  $F_{MSY}$ =0.20 qtr<sup>-1</sup> with percentages of maximum YPR and SPR of 99% and 22%. Note that the value of  $F_{MSY}$  was effectively determined by the assumed value of steepness h=0.9 and the value of adult natural mortality M=0.35. The value of  $F_{MAX}$  was  $F_{MAX}$ =0.25 qtr<sup>-1</sup> with percentages of maximum YPR and SPR of 100% and 18%. The value of  $F_{0.1}$  was  $F_{0.1}$ =0.10 qtr<sup>-1</sup> with percentages of maximum YPR and SPR of 88% and 39%. The value of  $F_{MED}$  was  $F_{MED}$ =0.12 qtr<sup>-1</sup> with percentages of maximum YPR and SPR of 94% and 32%. Last, the value of  $F_{CRASH}$  was beyond the range of F values examined in the YPR program and was determined to be greater than 3, that is,  $F_{CRASH} > 3$  qtr<sup>-1</sup>.

Results of the  $B_{SPR}$  calculations indicated that  $B_{SPR}$  ranged from  $B_{10\%}=7578$  mt to  $B_{90\%}=77716$  mt, a ten-fold range (Table 4). Ratios of reference biomass to  $B_{MSY}$  ranged from 0.42 to 4.26 while ratios of reference yield to MSY ranged from 1.00 to 0.18.

The other biomass reference points fell within the range of  $B_{SPR}$  values examined (Tables 4 and 5). The value of  $B_{MSY}$  was  $B_{MSY}$ =18258 mt. The value of  $B_{MAX}$  was  $B_{MAX}$ =14694 mt with a ratio of reference yield to MSY of 0.99. The value of  $B_{0.1}$  was  $B_{0.1}$ =32746 mt with a ratio of reference yield to MSY of 0.91. The value of  $B_{MED}$  was  $B_{MED}$ =27242 mt with a ratio of reference yield to MSY of 0.96. Last, the nominal equilibrium value of  $B_{CRASH}$  was 0 mt because this reference point corresponded to population extinction.

# References

Beverton, R., and Holt, S. 1957. On the dynamics of exploited fish populations. Chapman and Hall, London. Fascimile reprint, 1993.

Brooks, E., Powers, J., and Cortes, E. 2010. Analytical reference points for age-structured models: application to data-poor fisheries. ICES J. Mar. Sci. 67:165-175.

Courtney D., and Piner, K. 2010. Age Structured Stock Assessment of North Pacific Swordfish (*Xiphias gladius*) with Stock Synthesis under a Two Stock Scenario. ISC BILLWG Workshop, 15-22 April 2010, Hokkaido University, School of Fisheries Sciences, Hakodate, Hokkaido, Japan. ISC/10/BILLWG-1/01.

Gabriel, W., Sissenwine, M., and Overholtz, W. 1989. Analysis of spawning stock biomass per recruit: an example for Georges Bank haddock. N. Am. J. Fish. Manag. 9:383-391.

Gulland, J., and Boerema, L. 1971. Scientific advice on catch levels. Fish. Bull. 71:325-335.

Mace, P., and Sissenwine, M. 1993. How spawning per recruit is enough? P. 101-118. In S. Smith, J. Hunt, and D. Rivard [Eds.] Risk evaluation and biological reference points for fisheries management. Can. Spec. Publ. Fish. Aquat. Sci. 120.

Myers, R., Bowen, K., and Barrowman, N. 1999. Maximum reproductive rate of fish at low population sizes. Can. J. Fish. Aquat. Sci. 56:2404-2419.

Sissenwine, M., and Shepherd, J. 1987. An alternative perspective on recruitment overfishing and biological reference points. Can. J. Fish. Aquat. Sci. 44:913-918.

Table 1. Life history and fishery parameters by quarter used in biological reference point calculations.

					Fraction						Fraction
					Mature at						Mature at
			Spawning	Fishery	Age and				Spawning	Fishery	Age and
	Fishery	Natural	Stock	Catch	Actively		Fishery	Natural	Stock	Catch	Actively
Age (in	Selectivity		Weights at		Spawning	Age (in	Selectivity				Spawning
quarters)	at Age		-	at Age	by Quarter	quarters)		at Age	Age	-	by Quarter
quarters)	0.00	at Age 0.100	Age 5.5	5.5		 41	at Age 0.98	0.088	173.41	at Age 173.41	
2	0.00	0.100	8.23	8.23	0	41	0.98	0.088	175.41	175.41	0
3	0.04	0.098	11.5	8.25 11.5	0	42	0.99	0.088	175.27	175.27	0
4	0.00	0.097	15.27	15.27	0.16	44	0.97	0.088	177.03	178.69	1
5	0.09	0.096	19.49	19.49	0.16	44	0.98	0.088	178.09	178.09	0
6	0.19	0.095	24.1	24.1	0	 45	0.98	0.088	180.25	180.25	0
7	0.28	0.094	24.1	24.1	0	 40	0.99	0.088	181.72	181.72	0
8	0.34	0.093	34.28	34.28	0.53	 47	0.98	0.088	183.11	183.11	1
9 10	0.51	0.092	39.72	39.72	0	 49 50	0.98	0.088	185.66	185.66	0
-	0.62	0.092	45.33	45.33	0	50	0.99	0.088	186.82	186.82	0
11	0.65		51.06	51.06		51	0.98		187.91	187.91	0
12	0.64	0.091	56.85	56.85	0.82		0.98	0.088	188.94	188.94	
13	0.74	0.091	62.68	62.68	0	53	0.98	0.088	189.91	189.91	0
14	0.82	0.090	68.49	68.49	0	 54	1.00	0.088	190.83	190.83	0
15	0.81	0.090	74.26	74.26	0	 55	1.00	0.088	191.68	191.68	0
16	0.80	0.090	79.96	79.96	0.93	 56	1.00	0.088	192.49	192.49	1
17	0.86	0.090	85.57	85.57	0	 57	1.00	0.088	193.25	193.25	0
18	0.91	0.090	91.07	91.07	0	 58	1.00	0.088	193.96	193.96	0
19	0.88	0.089	96.43	96.43	0	59	1.00	0.088	194.63	194.63	0
20	0.88	0.089	101.66	101.66	0.97	60	1.00	0.088	195.26	195.26	1
21	0.91	0.089	106.72	106.72	0	 61	1.00	0.088	195.86	195.86	0
22	0.95	0.089	111.63	111.63	0	62	1.00	0.088	196.41	196.41	0
23	0.92	0.089	116.36	116.36	0	 63	1.00	0.088	196.94	196.94	0
24	0.93	0.089	120.92	120.92	0.98	64	1.00	0.088	197.43	197.43	1
25	0.94	0.089	125.31	125.31	0	 65	1.00	0.088	197.89	197.89	0
26	0.97	0.089	129.52	129.52	0	66	1.00	0.088	198.32	198.32	0
27	0.94	0.089	133.55	133.55	0	 67	1.00	0.088	198.73	198.73	0
28	0.95	0.089	137.4	137.4	0.99	68	1.00	0.088	199.11	199.11	1
29	0.96	0.088	141.09	141.09	0	69	1.00	0.088	199.47	199.47	0
30	0.98	0.088	144.6	144.6	0	70	1.00	0.088	199.81	199.81	0
31	0.96	0.088	147.95	147.95	0	71	1.00	0.088	200.12	200.12	0
32	0.96	0.088	151.14	151.14	0.99	72	1.00	0.088	200.42	200.42	1
33	0.97	0.088	154.17	154.17	0	73	1.00	0.088	200.7	200.7	0
34	0.98	0.088	157.05	157.05	0	74	1.00	0.088	200.96	200.96	0
35	0.96	0.088	159.79	159.79	0	75	1.00	0.088	201.2	201.2	0
36	0.97	0.088	162.38	162.38	0.99	76	1.00	0.088	201.43	201.43	1
37	0.97	0.088	164.84	164.84	0	77	1.00	0.088	201.65	201.65	0
38	0.99	0.088	167.16	167.16	0	78	1.00	0.088	201.85	201.85	0
39	0.97	0.088	169.36	169.36	0	79	1.00	0.088	202.04	202.04	0
40	0.98	0.088	171.45	171.45	1	80	1.00	0.088	202.22	202.22	1

				Ratio of	Ratio of	Percent of	Percent of
Reference	F Value			Reference F	Reference F	Maximum	Maximum
Point	(quarter <sup>-1</sup> )	YPR (kg)	SPR (kg)	to Adult M	to FMSY	YPR	SPR
F10%	0.478	16.32266	12.32363	5.5	2.4	96%	10%
F20%	0.228	16.9062	24.6459	2.6	1.1	100%	20%
F30%	0.139	16.1654	36.98484	1.6	0.7	95%	30%
F40%	0.092	14.72539	49.31412	1.1	0.5	87%	40%
F50%	0.063	12.83549	61.62396	0.7	0.3	76%	50%
F60%	0.043	10.61927	73.97194	0.5	0.2	63%	60%
F70%	0.028	8.17547	86.28718	0.3	0.1	48%	70%
F80%	0.016	5.55123	98.64665	0.2	0.1	33%	80%
F90%	0.007	2.81454	110.9547	0.1	0.0	17%	90%
F100%	0	0	123.2254	0.0	0.0	0%	100%

Table 2. Fishing mortality reference points based on percentages of the spawning potential ratio  $F_{SPR}$ .

Table 3. Fishing mortality reference points to maximize yield ( $F_{MSY}$ ), to maximize yield per recruit ( $F_{MAX}$ ), to achieve 10% of the maximum marginal increase in yield per recruit ( $F_{0.1}$ ), and to achieve the median survival ratio ( $F_{MED}$ ).

				Ratio of	Ratio of	Percent of	Percent of
Reference	F Value			Reference F	Reference F	Maximum	Maximum
Point	(quarter <sup>-1</sup> )	YPR (kg)	SPR (kg)	to Adult M	to FMSY	YPR	SPR
FMSY	0.202	16.82486	27.34281	2.3	1.0	99%	22%
FMAX	0.255	16.92986	22.32999	2.9	1.3	100%	18%
F0.1	0.097	14.94132	47.71559	1.1	0.5	88%	39%
FMED	0.125	15.8679	39.9758	1.4	0.6	94%	32%

					Ratio of	
	Spawning		Fishery	Ratio of	Reference	Ratio of
Reference	Biomass	Recruitment	Yield	Reference B	Recruitment	Reference
Point	(mt)	(000s)	(mt)	to BMSY	to RMSY	Yield to MSY
B10%	7578.1	614.9	10037.3	0.42	0.92	0.89
B20%	16340.7	663.0	11209.1	0.89	0.99	1.00
B30%	25115.1	679.1	10977.3	1.38	1.02	0.98
B40%	33882.6	687.1	10117.5	1.86	1.03	0.90
B50%	42636.3	691.9	8880.6	2.34	1.04	0.79
B60%	51417.1	695.1	7381.3	2.82	1.04	0.66
B70%	60174.6	697.4	5701.4	3.30	1.04	0.51
B80%	68963.6	699.1	3880.8	3.78	1.05	0.35
B90%	77716.0	700.4	1971.4	4.26	1.05	0.18
B100%	86441.8	701.5	0	4.73	1.05	0.00

Table 4. Biomass reference points based on percentages of the spawning potential ratio B<sub>SPR</sub>.

Table 5. Biomass reference points to maximize yield (B <sub>MSY</sub> ), to maximize yield per recruit
$(B_{MAX})$ , to achieve 10% of the maximum marginal increase in yield per recruit $(B_{0.1})$ , and to
achieve the median survival ratio $(B_{MED})$ .

					Ratio of	
	Spawning		Fishery	Ratio of	Reference	Ratio of
Reference	Biomass	Recruitment	Yield	Reference B	Recruitment	Reference
Point	(mt)	(000s)	(mt)	to BMSY	to RMSY	Yield to MSY
BMSY	18258.5	667.8	11235.0	1.00	1.00	1.00
BMAX	14693.8	658.0	11140.3	0.80	0.99	0.99
B0.1	32745.8	686.3	10253.8	1.79	1.03	0.91
BMED	27242.0	681.5	10813.4	1.49	1.02	0.96

Figure 1. Time series of estimates of recruits per spawning biomass (kg) of WCPO swordfish taken from Courtney and Piner (2010).

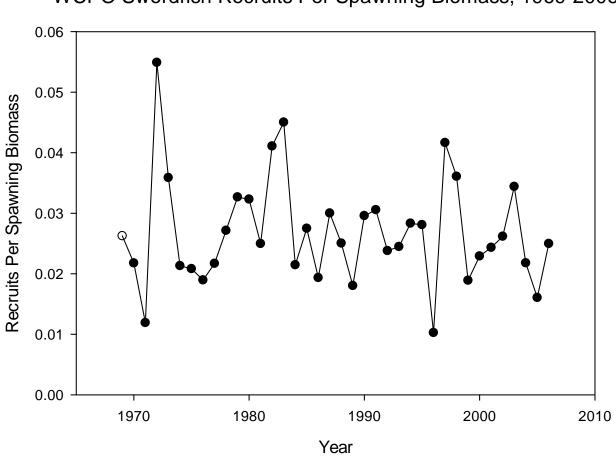
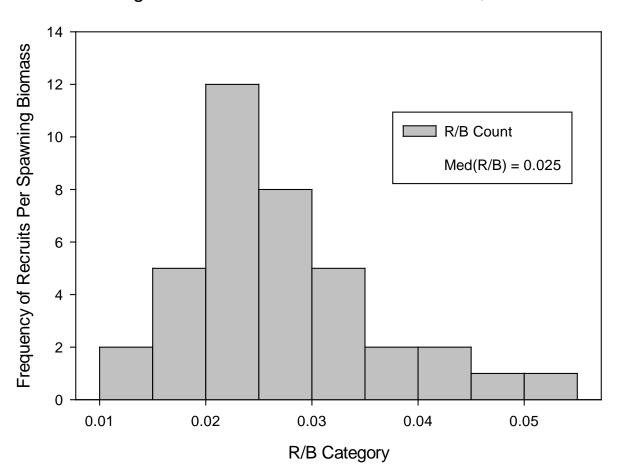




Figure 2. Empirical distribution of estimates of recruits per spawning biomass (kg) of WCPO swordfish taken from Courtney and Piner (2010).



Histogram of WCPO Swordfish R/B Values, 1969-2006

Figure 3. Scatterplot of estimates of recruitment (thousands) and spawning biomass (kg) of WCPO swordfish and the associated value of  $F_{MED}$  (solid line) based on the median of the empirical R/B distribution.

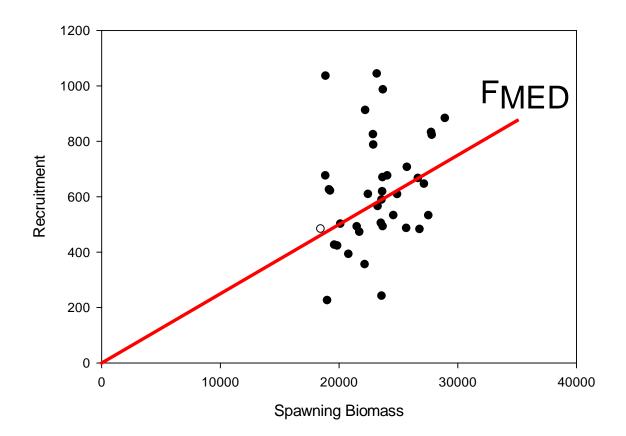


Figure 4. Spawning biomass per recruit (kg) of WCPO swordfish as a function of fishing mortality along with fishing mortality rates to achieve a fixed percentage of maximum spawning potential (F<sub>%MSP</sub>) and replacement fishing mortality (F<sub>MED</sub>).

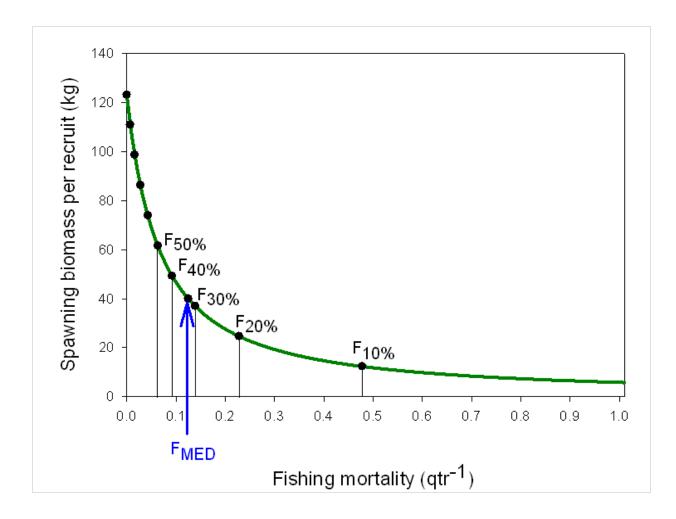


Figure 5. Yield per recruit (kg) of WCPO swordfish as a function of fishing mortality along with fishing mortality rates to produce 10% of the maximal increase in YPR ( $F_{0.1}$ ), the maximum sustainable yield ( $F_{MSY}$ ), and the maximum yield per recruit ( $F_{MAX}$ ).

