Preliminary runs of future stochastic projections from outputs of SS for the North Pacific albacore

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Introduction

This document shows preliminary results of future projections from outputs by Stock Synthesis (SS) with different settings. The stochastic projections with SS outputs are conducted by specific software coded by R, which have been used in PBF-WG for the stock assessment of Pacific bluefin tuna (Ichinokawa et al. 2007, Ichinokawa et al. 2008). By over-viewing recruitment and harvesting scenarios and other options implemented by this software, availability of this software and further requirements could be discussed in order to obtain reasonable results on stochastic projections and reference points for the next stock assessment of North Pacific albacore.

Materials and method

R-code for stochastic projections with SS outputs

Appendix I describes algorithms to conduct stochastic projections with the R-code, and available options for future scenarios. In addition, method for calculating suites of reference points is described.

Data and SS model

SS output for the preliminary calculation of future projections are derived from preliminary data and control files for SS, which was distributed before this WG. Note that projection results shown in documents are only for testing the R-code and available options, which does not reflect any actual results of stock assessments for the North Pacific albacore.

Configurations for future projections

Options available with the software are shown in Table 1. Possible base options for the next stock assessment are underlined in the table, and used as a base case in this document (Table 2). Of course, final choices of those options will be done after enough investigations of final stock assessment results, which will apper next WG. Also, note that stochastic future projections for this document are conducted with 500 times stochastic simulations from point estimates, not from bootstrap estimates, because there wasn't enough time to conduct bootstraps. In the next WG for the final stock assessment with SS, stochastic simulations will be conducted from bootstrap estimates with enough number of replications.

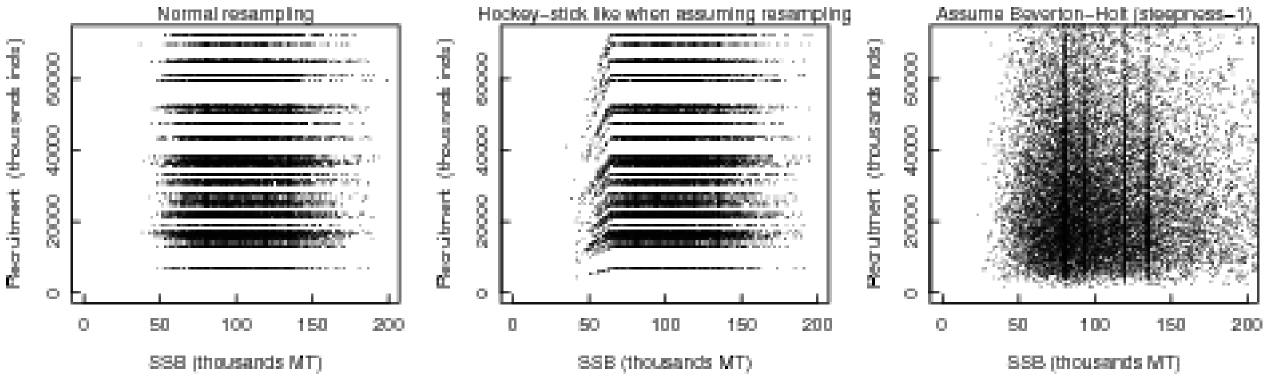

Figure 1: Examples of future recruitment scenarios. Left panel: simple re-sampling of past recruitments during 1965−2008. Middle panel: assuming hockey-stick type recruitments based on re-sampling of past recruitments. In this example, the threshold of SSB is set to be lower 20% of the historically observed SSB. Right panel: beverton-Holt spawner-recruitment relationship and log-normal error. Parameters of the S-R curve are inherited from the estimates in SS. In this example, steepness is fixed as 1 in SS.

Test runs and examples

Validation of the R-code with deterministic run of future projections by SS

Average numbers at age (Fig. 3) and partial catch by fisheries (Fig. 4) estimated from stochastic future projections implemented by the R-code are compared with those calculated by deterministic projections implemented by SS. Those runs follow the setting of run 0, except for time period of current F. This run (run 1) used F in 2008 as current F, instead of average F during 20006-2008 in the base case.

While average numbers at age (Fig. 3) are almost identical between the two results from the R-code and SS, partial catch by fisheries and estimated SSB in weight (Fig. 4) are slightly different especially in the catch by fleet 6 and SSB. The differences of the partial catch are caused by different assumptions of annual partial F by fleet between SS and this R-code. There are no clear description on the way to assume future partial F in SS, so that the R could not trace every procedures in SS. The difference in SSB is may because SS (>3.10a) calculate weight at terminal age from initial numbers at age older than the terminal age with assumption of M=0.2, but it shouln't cause the differences. Details in future projection by SS should be asked to Dr. Methot. In near future, the R-code should be modified to follow the way to calculate weight at terminal age as used in SS.

Results of preliminary runs testing available options

The effects of changes in the year to start stochastic projections (Fig. 5), the way to average current F (geometric or simple mean, Fig. 6), the time period to calculate current F (Fig. 7), recruitment scenarios (Fig. 8), harvesting scenarios (Fig. 9).

0.1 Discussion: configuration for future projection

See Table 1.

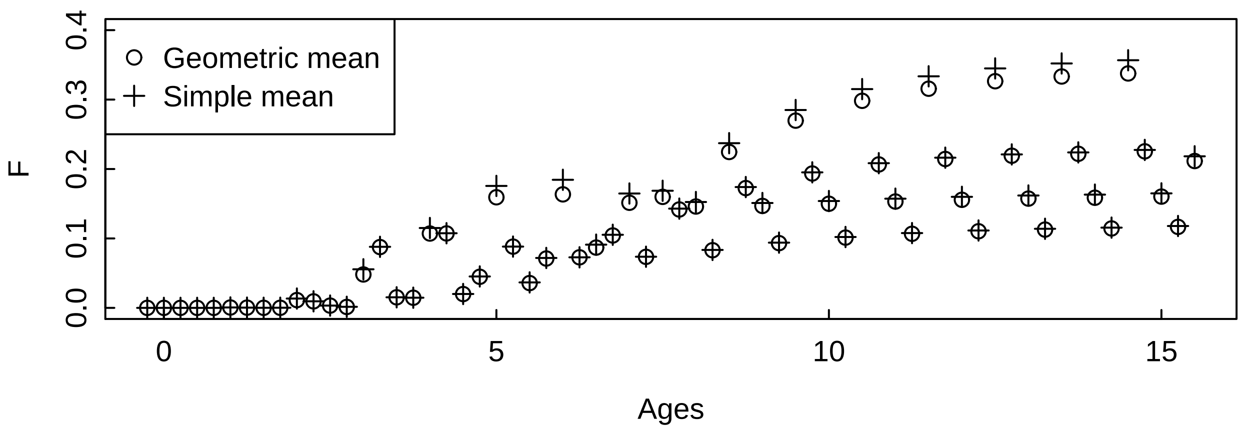

Figure 2: Differences of current F with simple and geometric averages

Table 1: Lists of possible options in settings of future projections for the stock assessment of North Pacific albacore

Start year	<u>S0</u>	2009
	S1	Before 2008 (depending on time period when recruitment deviations are estimated, and retrospective patterns)
Future recruitment	R0	Random re-sampling
	<u>R0-1</u>	Simple random re-sampling of past recruitments during the stock assessment period such as 1965-2008 (Fig. 1, left). The period for recruitment re-sampling depends on time period when recruitment deviations are estimated.
	R0-2	Assume high or low recruitment, taking a specific period when recruitments are relatively lower or higher. This is applied to the 2004 stock assessment.
	R0-3	Hockey-stick + random-resampling (Fig. 1, middle). See Appendix 0.4.4 for details.
	R1	Beverton-Holt + lognormal error (depending on estimated steepness in SS) (Fig. 1, left). Details are in Appendix 0.4.2.
	R2	Others
Harvesting scenarios	<u>H0</u>	Constant fishing mortality (current F $\times$ multiplier). F multiplier can be set by annual and fisheries.
	H1	Constant catch (some scenarios are not implemented yet.)
	H1-1	Constant catch by quarter. In this option, constant quarterly catch can be given.
	H1-2	Constant catch by fisheries and quarter. In this option, F at age should be calculated in the same procedure in SS. This is too messy.
	H2	combination of constant catch (H1-1 or H1-2) and F (H1). This is probably needed for adjust catch during the periods between last year of stock assessment period and the year when stock assessment is conducted. In this case, combination of H1-2 and H0 is ideal.
	H3	Constant F and catch capping. Basically, the projection is conducted by given F, but set the upper limit of annual catch by fisheries.
	H4	others
'Current' F period	<u>F0</u>	2006-2008 (Pick 3 years including the terminal year)
	F1	2005-2007 (Pick 3 years excluding the terminal year)
	F2	Others. It should be determined after retrospective patterns and SD of the terminal year's estimates.
'Current' F average	<u>A0</u>	Geometric mean (Fig. 2)
	A1	Simple mean (Fig. 2)
Number of bootstraps	<u>B0</u>	0 (tentatively in this document)
	B1	> 300 (recommended in the next stock assessment)
	B2	500 (in the 2004 stock assessment)
Number of simulations	<u>S0</u>	250 (tentatively in this document)
	S1	> 1 (depending on scenario B1)
	S2	1 (in the 2004 stock assessment)

Table 2: Settings for the preliminary calculations of future stochastic projection in this document

Run 0	S0	R0-1	H0	F0	A0	Base run in this document
Run 1	S0	R0-1	H0	F2 (F in 2008)	A0	For validation with SS results
Run 2	S1(2008)	R0-1	H0	F0	A0	Start from 2008
Run 3	S1(2007)	R0-1	H0	F0	A0	Start from 2007
Run 4	S0	R0-1	H0	F0	A1	Simple average
Run 5	S0	R0-1	H0	F1	A0	Drop terminal year for current F
Run 6	S0	R0-3	H0	F0	A0	Hockey-stick + random resampling
Run 7	S0	R1	H0	F0	A0	Beverton-Holt + lognormal error
Run 8	S0	R0-1	H1-1	F0	A0	Constant catch by quarter (10, 20, 10, 10 x thousands MT)

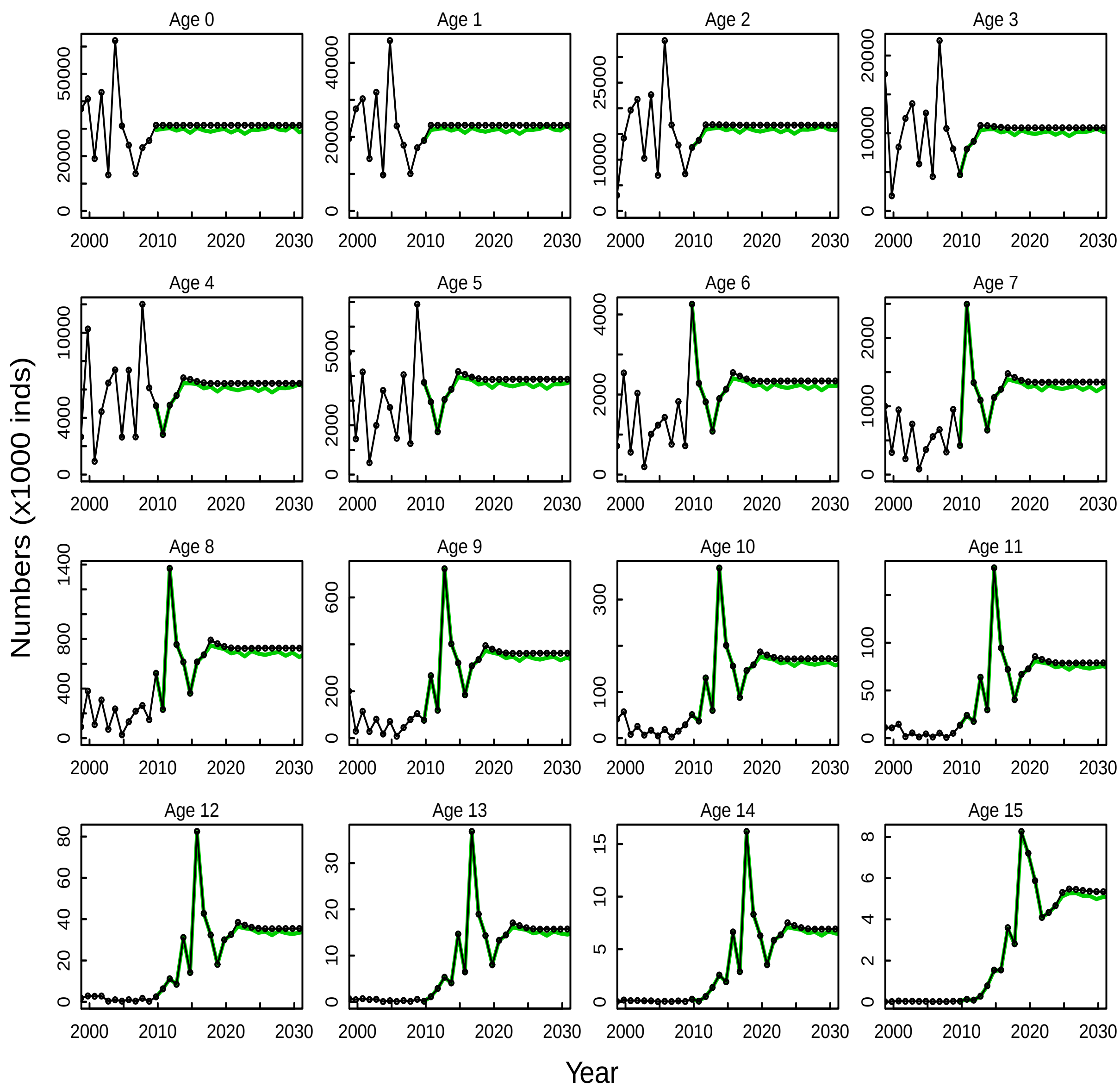

Figure 3: Comparison of numbers at age generated from stochastic projections by the R-code (average values, thick green lines) with those from a deterministic run by SS (lines with circles).

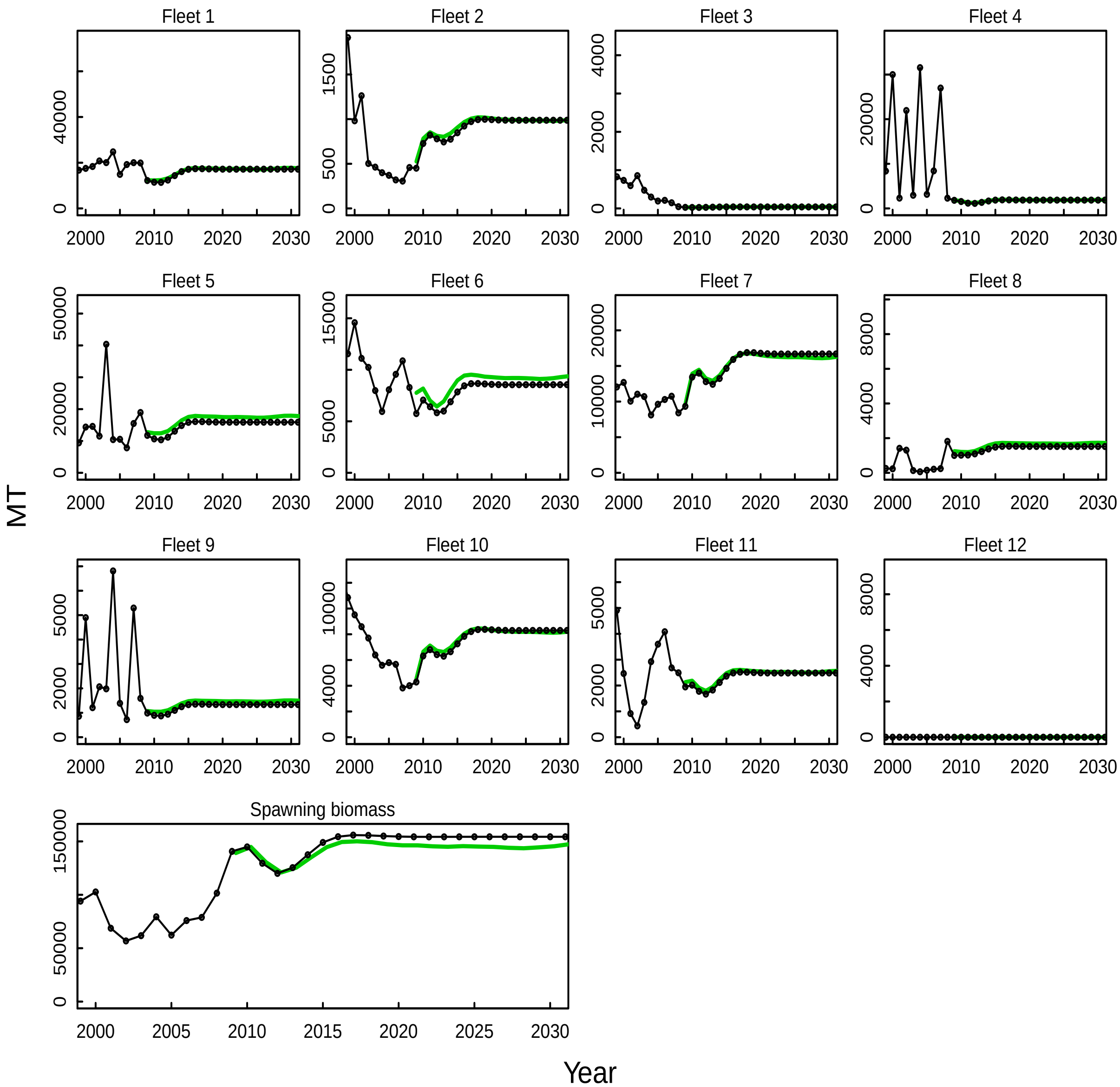

Figure 4: Comparison of catch in weight (MT) by fleets (figures titled by F1 to F12) and spawning biomass (MT) generated from stochastic projections by the R-code (thick lines) with those from a deterministic run by SS (lines with circles).

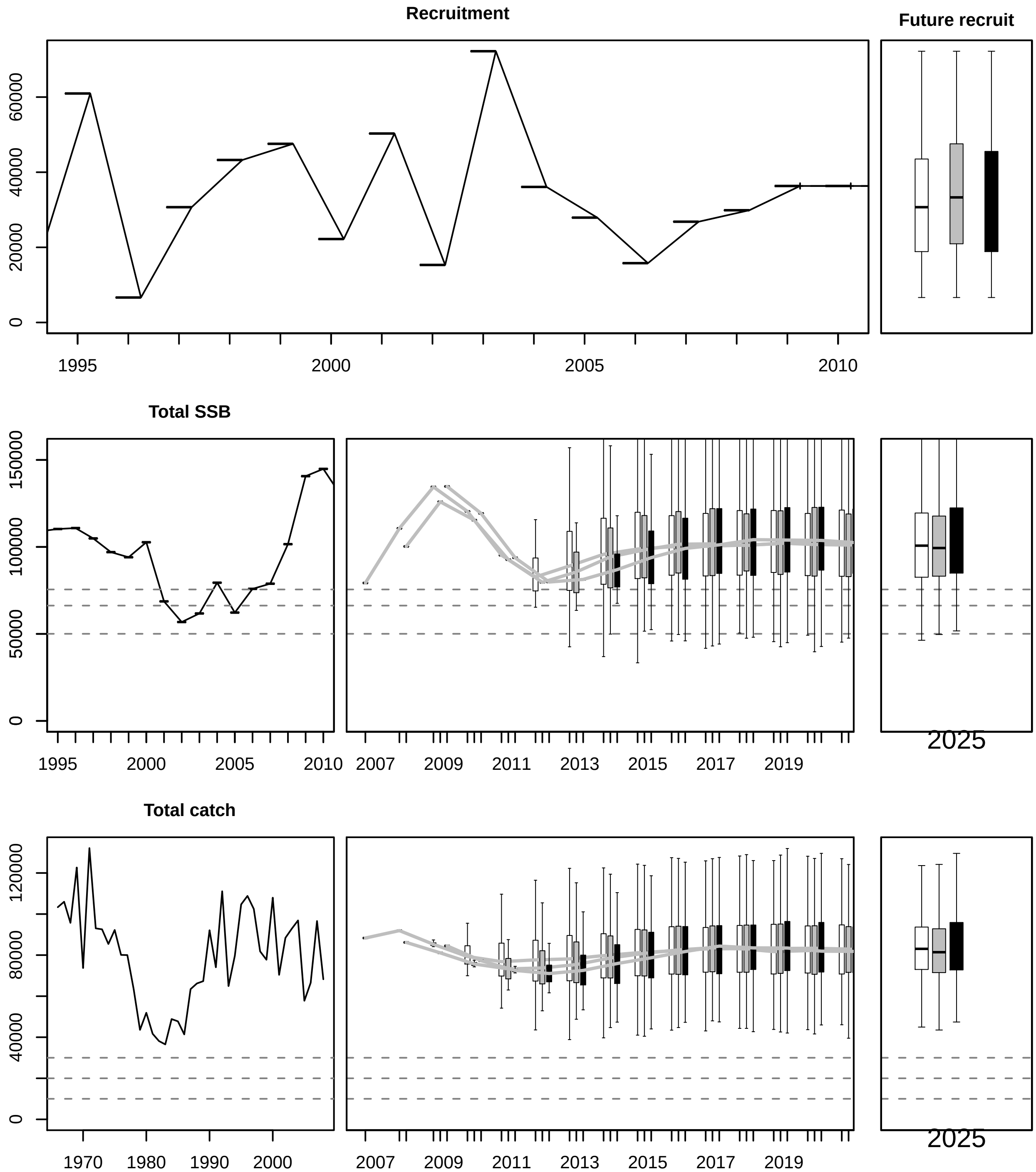

Figure 5: The start years for future projections are assumed as 2009 (black, run 0), 2008 (gray, run 2) and 2007 (white, run3).

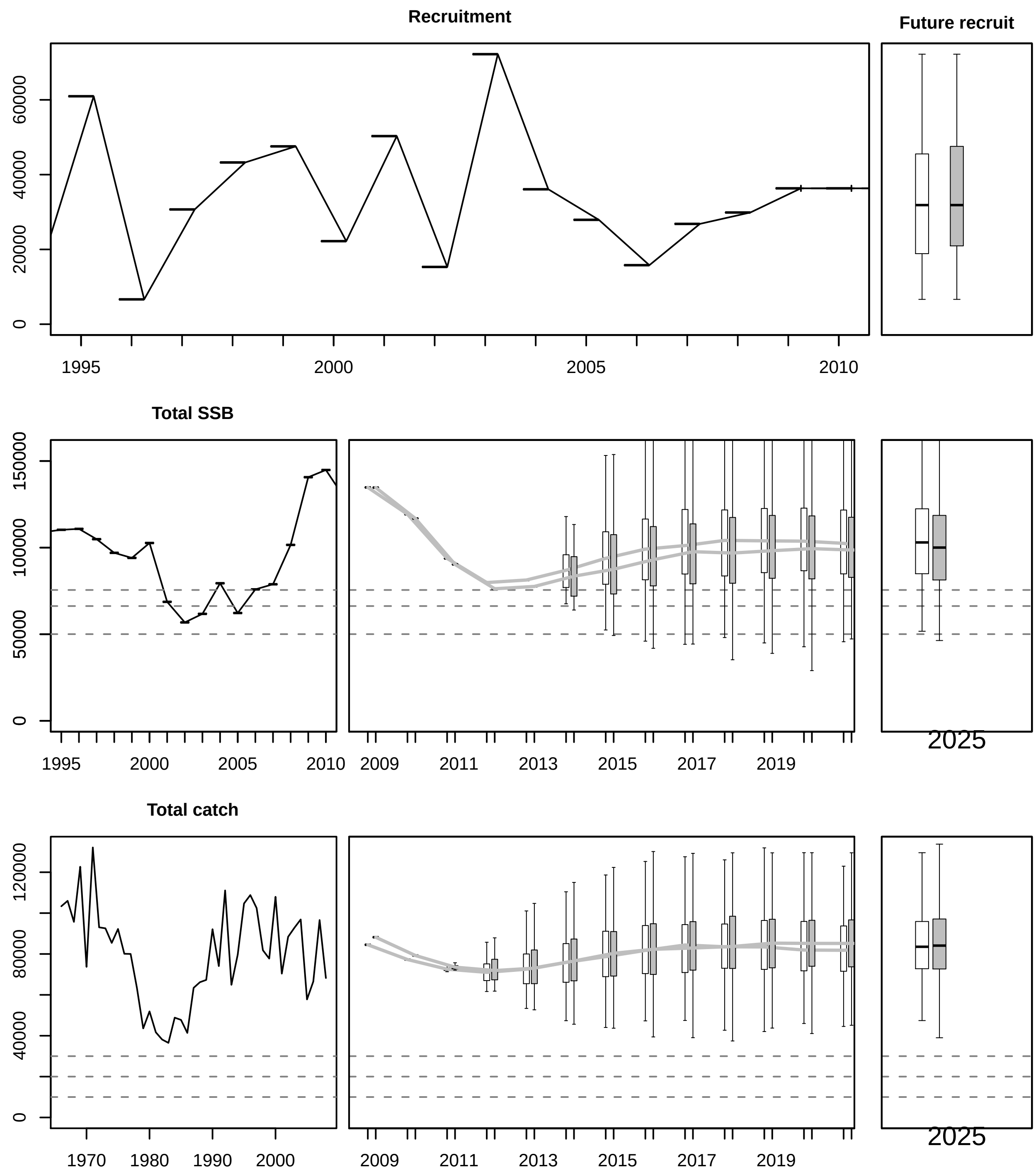

Figure 6: The way for averaging current F through 2006-2008: geometric (white, run 0) and simple (gray, run 4) means

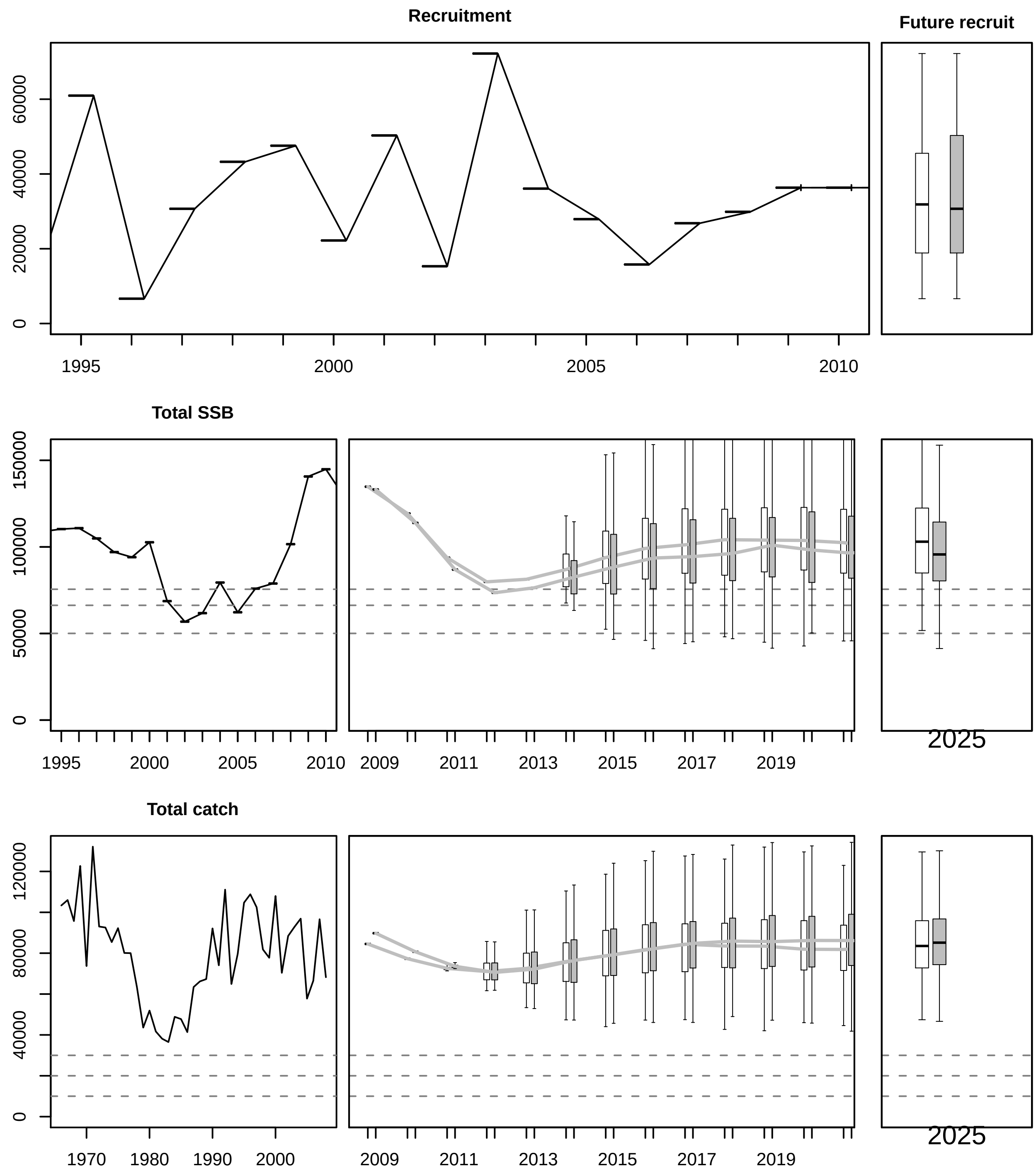

Figure 7: Different current F period between 2006-2008 (white, run 0) and 2005-2007 (gray, run 5)

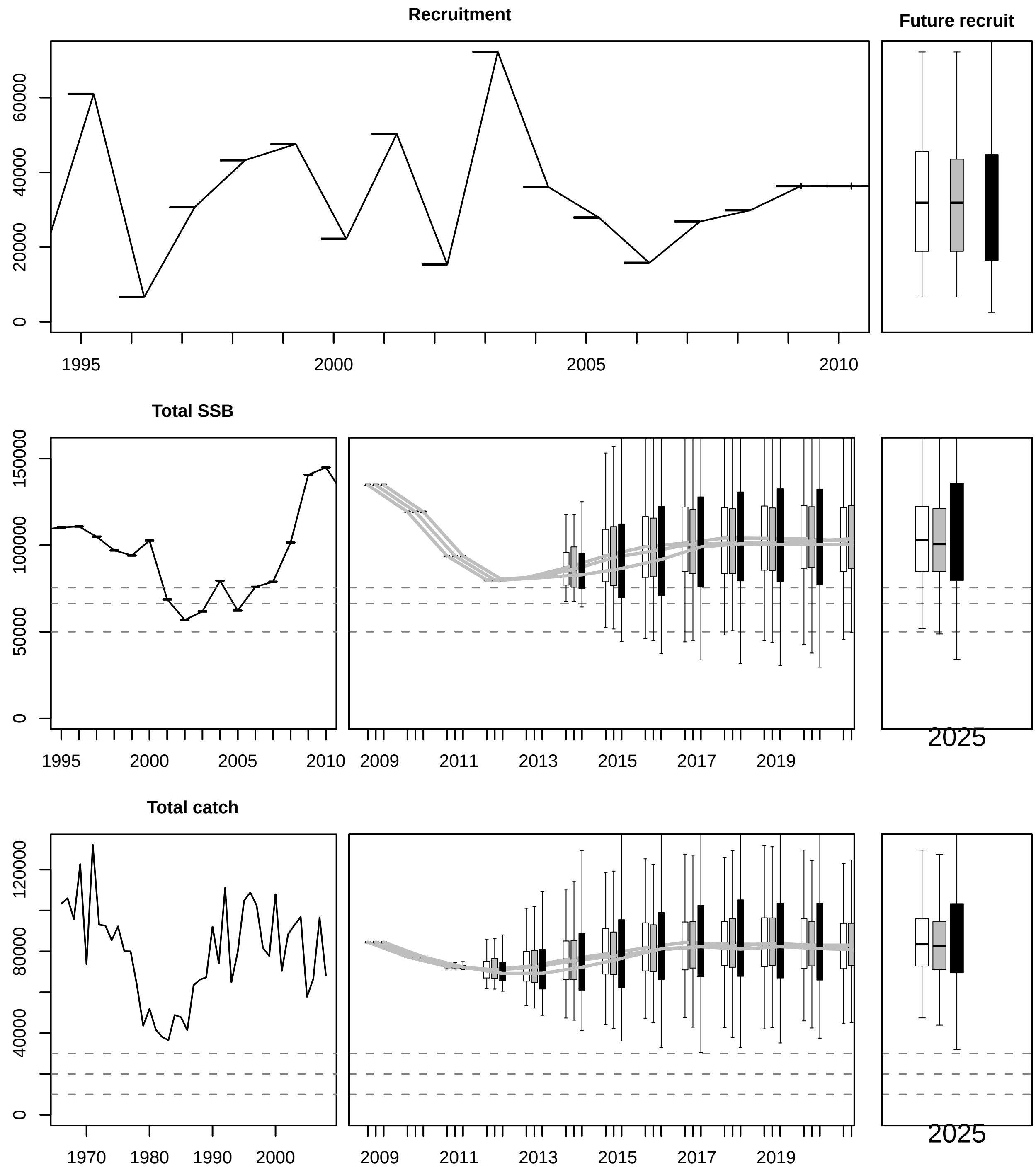

Figure 8: Assuming different future recruitment scenarios. Simple re-sampling of past recruitments during 1965 to 2008 (run0, white). Assuming hockey-stick type recruitments based on re-sampling of past recruitments (run 6, gray). Beverton-Holt assumed in SS and log-normal error (run 7, black). Same settings as shown in Fig. 1.

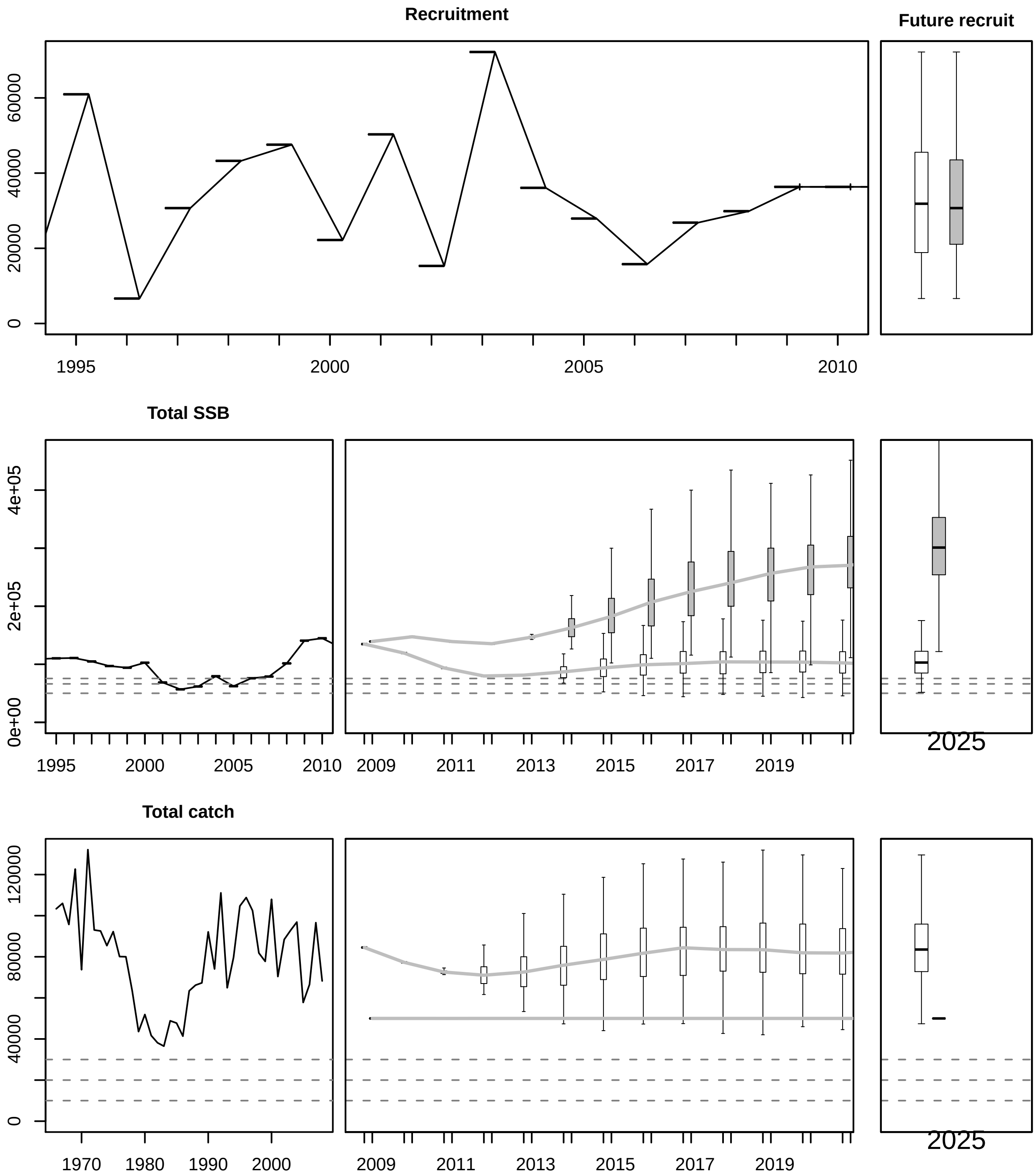

Figure 9: Constant F (run 0) & constant catch (run 8) scenarios. Note that the number of simulations in the constant catch scenario is 200 because of shortage for the calculation time.

Appendix: Algorithm for calculation of stochastic future projections and suites of reference points from Stock Synthesis outputs (draft)

0.2 Introduction

Stock Synthesis (SS) is currently one of the most commonly used software for stock assessment. The software provides statistical framework for calibration of a population dynamics model using a diversity of fishery and survey data (Methot, 2005). SS can also provide outlooks for future population dynamics with deterministic projections from the estimated parameters during the stock assessment period. In the future projection phase, SS provide averages of future statistics such as spawning biomass (SSB) and standard deviations of the future statistics. The standard deviations are estimated by Hessian-matrix under normal approximation (Maunder et al., 2006) or by MCMC. Therefore, the estimated future statistics (such as future SSB) in SS are assumed to distribute normally, and not reflect future stochastic uncertainty.

On the other hand, stochastic future projections are sometimes important for evaluating current stock status and future management procedures. Simulation-based reference points of F_{ssb} (Conser et al. 2006; Ichinokawa et al., 2010) are used as an interim reference point for this stock for North Pacific albacore stock. Future stochastic projections for the stock assessment of Pacific bluefin tuna in ISC have been conducted with specific software coded by R (cf. Ichinokawa et al., 2008). Suites of reference points are also produced with this software (cf. Kai et al. 2010).

This appendix made detailed description on algorithms to conduct stochastic future projections and reference points from outputs by Stock Synthesis with the R-code.

0.3 Population dynamics

0.3.1 Definition of parameters and dimension for population dynamics

Table 1 shows definition of parameters used in this document. Basically, parameters estimated during the stock assessment phase are expressed by lower cases; those during future projection phase are by upper cases. For example, numbers at age a ($0 \leq a \leq Amax$) and time t at the bth bootstrap run during the stock assessment period is expressed as $n^b_{a,t}$, while those in the future projection period are expressed by $N^{b,k}_{a,t}$. The superscript of b, representing the bth bootstrap results, can range from 0 to the number of bootstrap iterations of X_b. The 0th iteration represents maximum likelihood estimation.

Since multiple season per year can be assumed in SS, estimated parameters in SS include the dimension of season ($0 \leq s \leq nseason$). However, because description of the additional dimension make the R-code complicated, population dynamics used in the future projections didn't include the dimension of season. Instead, the time series is expanded by year and season to use year interval of $i = 1/nseason$. For example, time series from 1952 to 2006 with 4 seasons can be expressed by $t = 1952, 1952.25, 1952.5, ..., 2005.5, 2005.75,$ and age classes from 0 to 20 years old are by $a = 0, 0.25, 0.5, ..., 19.5, 19.75$. Then, estimates in SS of $n_{a,s,t}$ (numbers at age at the beginning of the

Table 3: Parameters used in this document

Parameter	Description
$n^b_{a,t}$, $N^{k,b}_{a,t}$	Population numbers at beginning of season
$f^b_{a,t}$, $F^{k,b}_{a,t}$	Instantaneous rate of fishing mortality during the stock assessment period (lower case) and future projection period (upper case)
$c^b_{a,t}$, $C^{k,b}_{a,t}$	Catch at age during the stock assessment period (lower case) and future projection period (upper case)
M_a	Instantaneous rate of natural mortality (/yr)
w_a	Weight at age at the beginning of season
w_{af}	Weight at age by fleet at the beginning of season
Q_a	Maturity rates at age
Fcur_a	Current F, which is produced by averaging a current F period
PFcur_{af}	Partial F corresponding with Fcur_a. $\sum_{k=1}^{nfleet} \mathrm{PFcur}_{af} = \mathrm{Fcur} = a$
b^b_t,B^b_t	Stock biomass at the beginning of the year
ssb^b_t,SSB^b_t	SSB spawning stock biomass at the beginning of the spawning season
PTC^b_t,TC^b_t	Total catch by fleet, and total catch in weight
t_1, t_2	Generic parameters specifying a range of time period from t_1 to t_2 (Table 2)
$nseason$	Number of seasons per year (inherited from SS)
$nfleet$	Number of fisheries (inherited from SS)
i	Time interval = 1/qt
A_{min},A_{max}	Minimum and maximum age considered in the stock assessment model and future projection. Plus groups are included in the age of A_{max}. Recruitments are assumed to occur at $a = 0$.
X_s	Number of stochastic simulations. $k = 1, 2, ..., X_s$.
X_b	Number of bootstrap iterations. $b = 0, 1, 2, ..., X_s$. The 0th iteration represents point estimates.

time time interval t),$c_{a,s,t}$ (catch at age of age a during the time interval t), and $c_{a,s,t,f}$ (partial catch at age by fisheries) are rewritten as the following equations.

$$n_{a+(s-1)/qt,t+(s-1)/qt} = \begin{cases} n_{a,s,t} & \text{if} \quad n_{a,s,t} > 0 \\ \mathrm{NA} & \text{if} \quad n_{a,s,t} = 0 \end{cases} \tag{1}$$

$$c_{a+(s-1)/qt,t+(s-1)/qt} = \begin{cases} c_{a,s,t} & \text{if} \quad n_{a,s,t} > 0 \quad and \quad a < A_{max} \\ \frac{1}{nseason} \sum_{s=1}^{nseason} c_{a,s,t} & \text{if} \quad n_{a,s,t} > 0 \quad and \quad a = A_{max} \\ \mathrm{NA} & \text{if} \quad n_{a,s,t} = 0 \end{cases} \tag{2}$$

$$c_{a+(s-1)/qt,t+(s-1)/qt,f} = \begin{cases} c_{a,s,t,f} & \text{if} \quad n_{a,s,t,f} > 0 \quad and \quad a < A_{max} \\ \frac{1}{nseason} \sum_{s=1}^{nseason} c_{a,s,t,f} & \text{if} \quad n_{a,s,t} > 0 \quad and \quad a = A_{max} \\ \mathrm{NA} & \text{if} \quad n_{a,s,t} = 0 \end{cases} \tag{3}$$

’NA’ stands for a blank element, where corresponding population does not exist. Also, since all elements of $n_{a,t}, c_{a,t}, c_{a,t,f}$ can’t be filled with the above equation, remaining elements are filled by ’NA’. Those blank columns expressed by ’NA’ are ignored int the following calculations. Note that, in the case of catch at age, catch of terminal ages are assumed to be an average through the season.

0.3.2 Current F

Current F (Fcur_a) is defined from the parameters estimated in the stock assessment period, numbers at age ($n^b_{a,t}$) and catch at age ($c^b_{a,t}$). First, $f^b_{a,t}$ is estimated by solving the following catch equation.

$$c^b_{a,t} = \frac{f_{a,t}}{f_{a,t} + M_a}(1 - \exp(-f_{a,t} - M_a))n_{a,t} \tag{4}$$

Given the time range representative to 'current' as $t_1 \leq t < t_2$, the current F of Fcur_a is calculated by averaging F at age during the time range. In averaging F at age through the current F period, two options can be selected: simple or geometric average. In using simple average, the following equation is used.

$$\mathrm{Fcur}_a = \frac{1}{\text{number of years}} \sum^{t_1 \leq t < t_2} f_{a,t} \tag{5}$$

Otherwise, geometric mean can also be available.

$$\mathrm{Fcur}_a = \exp\left(\sum^{t_1 \leq t \leq t_2} \log f_{a,t} \frac{1}{\text{number of years}}\right) \tag{6}$$

Current F by fleet ($\mathrm{Fcur}_{a,f}$) is also calculated as following equations using partial catch at age ($c_{a,t,f}$).

$$\mathrm{Fcur}_{a,f} = \mathrm{Fcur}_a \mathrm{PFcur}_{a,f} \tag{7}$$

where

$$\mathrm{PFcur}_{a,f} = \frac{1}{\text{number of years}} \sum^{t_1 \leq t \leq t_1} \frac{c_{a,t,f}}{c_{a,t}} \tag{8}$$

The parameter of $\mathrm{PFcur}_{a,f}$ is partial catch ratio assumed in the current F, which is utilized in calculating partial catch in weight by fleet in future. Note that the $\mathrm{PFcur}_{a,f}$ is calculated by using simple average, even though the option of 'geometric mean' is used. This is because zero partial fishing mortality appears frequently, which cause inability to calculate geometric mean.

0.3.3 Population dynamics in future

Numbers at age $N^{k,b}_{a,y}$ in the future period ($t_1 \leq t \leq t_2$) are calculated as the following equations.

$$N^{k,b}_{a,t_1} = \begin{cases} R^{k,b}_t & a = 0 \quad \text{and} t = \text{recruitment timing} \\ 0 & a = 0 \quad \text{and} t \neq \text{recruitment timing} \\ n^b_{a,t_1} & \end{cases} \tag{9}$$

$$N^{k,b}_{a,t} = \begin{cases} R^{k,b}_t & a = 0 \quad \text{and} \quad t = \text{recruitment timing} \\ 0 & a = 0 \quad \text{and} \quad t \neq \text{recruitment timing} \\ N^{k,b}_{a-i,t-i} \exp(-F_{a-i} + -M_{a-i}) & 0 < a < A_{max} \\ N^{k,b}_{a-i,t-i} \exp(-F_{a-i} + -M_{a-i}) + N^{k,b}_{a,t-i} \exp(-F_a + -M_a) & a = A_{max} \end{cases} \tag{10}$$

Future recruitment of R_t and fishing mortality at age of F_a are determined by optional scenarios of recruitment (see section 0.4) and harvesting (see section 0.5).

Future statistics of total biomass ($B_t^{k,b}$) and SSB ($\mathrm{SSB}_t^{k,b}$) are calculated.

$$B_y^{k,b} = \sum_{a=0}^{A_{max}} w_a N_{a,t}^{k,b} \tag{11}$$

$$\mathrm{SSB}_y^{k,b} = \sum_{a=0}^{A_{max}} w_a Q_a N_{a,t}^{k,b} \tag{12}$$

weight at age, w_a, is derived from SS results (column specified by 'Wt_Beg'). Total catch ($\mathrm{TC}_t^{k,b}$, in weight) and total partial catch by fisheries ($\mathrm{PTC}_t^{k,b}$, in weight) are also calculated.

$$\mathrm{TC}_t^{k,b} = \sum_{f=1}^{nfleet} \mathrm{PTC}_{t,f}^{k,b} = \sum_{f=1}^{nfleet} w_{a,f} \sum_{a=0}^{A_{max}} \frac{F_a \mathrm{PFcur}_{a,f}}{F_a + M_a}(1 - exp(-F_a + M_a))N_{a,t}^{k,b} \tag{13}$$

Fishery specific weight at age (w_{af}) is used in calculating TC and PTC. The parameter of w_{af} is derived from SS results (column specified by 'Selwt' in Report.sso).

0.4 Recruitment scenarios

Following scenarios are available for future recruitments.

0.4.1 Simple resampling (REC.highlow)

This scenario assumes future recruitments as random re-samplings from recruitments estimated in the stock assessment periods specified by $t_1 \leq t \leq t_2$.

$$R_t^{k,b} = \begin{cases} \text{random draw from} \quad n_{0,t_1 \leq t \leq t_2} & t = \text{recruitment timing} \\ 0 & t \neq \text{recruitment timing} \end{cases} \tag{14}$$

Followings are required configurations in using this recruitment scenario.

year.lim(=NULL)	Give a vector with 2 elements of t_1 and t_2
qt(=1)	Assumed number of seasons per year (be consistent with SS configuration).
recruit.qt(=1)	Season when recruitments occur (be consistent with SS configuration). Currently, multiple recruitments per year can't be assumed with this R-code.

0.4.2 Deterministic + Log-normal error (REC.ss2)

Future recruitment is assumed to occur according to the Beverton-Holt relationship estimated in SS with random log-normal distributions. The parameters of the stock-recruitment relationship (h, S_0, R_0) and σ is basically derived from SS output.

$$R_t^{k,b} = \begin{cases} \hat{R}_t \exp\left(N(-\sigma^2/2, \sigma^2)\right) & t = \text{recruitment timing} \\ 0 & t \neq \text{recruitment timing} \end{cases} \tag{15}$$

where

$$\hat{R}_t = \frac{4hR_0\mathrm{SSB}_t}{S_0(1-h) + \mathrm{SSB}_t(5h-1)} \tag{16}$$

0.4.3 Assume empirical spawner-recruit relationship (REC.twophase)

This option is only for testing the effects of empirical spawner-recruit relationship observed in the Pacific bluefin tuna.

0.4.4 Options applied to all recruitment scenarios

In addition, hockey-stick type recruitments can be assumed for the all recruitment scenarios. The parameter of $\mathrm{SSB}_{threshold}$ should be specified with this option.

$$R'_t = \begin{cases} R_t \dfrac{\mathrm{SSB}_t}{\mathrm{SSB}_{threshold}} & \mathrm{SSB}_t \leq \mathrm{SSB}_{threshold} \\ R_t & \mathrm{SSB}_t > \mathrm{SSB}_{threshold} \end{cases} \tag{17}$$

0.5 Harvesting scenarios

0.5.1 Constant F (MP.CESqt3)

F_a is given by Fcur_a scaled by a multiplier, F_{multi}, determined by the optional settings described below.

$$F_a = F_{multi} * \mathrm{Fcur}_a \tag{18}$$

Fcur1(=NULL)	Give a vector with 2 elements of t_1 and t_2. The parameter of Fcur_a is calculated with this time duration. Fcur1 is used before the year to start regulation.
Fcur2(=NULL)	Give a vector with 2 elements of t_1 and t_2. The parameter of Fcur_a is calculated with this time duration. Fcur1 is used after the year to start regulation.
start.regulation(=2011)	Start year for regulation (switch F vectors from Fcur1 to Fcur2).
CES.multi(=1)	Simple multiplier to F_a (both for Fcur1 and Fcur2).
gm(=FALSE)	Use geometric mean (TRUE) or simple average (FALSE) for calculating Fcur_a.
CES.plus(=0)	Simple additional F to F_a.
CES.multi.year(=NULL)	Multiple vectors when changing multipliers to F_a by year. Even though the population dynamic consider season, this vector is given by year.
catch.capping(=NULL)	Set upper limit of annual partial catch (MT) by fleet. 'NULL' is not to use this option. If using this option, provide a list to specify target fisheries, upper limit of catch and reset timing.

0.5.2 Constant catch by season (MP.CHSfleet)

F_a is given by $\mathrm{Fcur}_a = \mathrm{PFcur}_{af}$ scaled by a multiplier determined by solving catch equation (eq. 13) to archieve given future catches in weight by season.

CHS.qt (=NULL)	Give a vector with *nsesaon* elements assigning catch in weight per quarter.
Fcur1(=NULL)	Give a vector with 2 elements of t_1 and t_2. The parameter of $\mathrm{Fcur}_a and \mathrm{PFcur}_{af}$ is calculated with this time duration. Fcur1 is used before the year to start regulation. In constant catch scenario, the estimated $\mathrm{Fcur}_a and \mathrm{PFcur}_{af}$ is scaled to adjust constant catch.
Fcur2(=NULL)	Give a vector with 2 elements of t_1 and t_2. The parameter of Fcur_a is calculated with this time duration. Fcur1 is used after the year to start regulation. In constant catch scenario, the estimated $\mathrm{Fcur}_a and \mathrm{PFcur}_{af}$ is scaled to adjust constant catch.
start.regulation(=2011)	Start year for regulation (switch F vectors from Fcur1 to Fcur2).
gm(=FALSE)	Use geometric mean (TRUE) or simple average (FALSE) for calculating Fcur_a.

0.5.3 Scenarios not implemented

- Constant catch by quarter and fleet
- Combination of constant catch (recent years) + constant F

0.6 Calculation of reference points

0.6.1 F_{ssb}

F_{ssb} is based on the probability that future SSB will fall below a given threshold of $\mathrm{SSB}_{threshold}$ at one or more years, during the assumed projection period ($t_1 \le t \le t_2$). The probability can be calculated from results of the stochastic projections.

$$\Pr\left[\mathrm{SSB}^{k,b}_{future} < \mathrm{SSB}^{b}_{threshold}|F\right] = \frac{1}{X_b X_s}\sum_{k=1}^{X_b X_s} || \min\left(\mathrm{SSB}^{k,b}_{t_1 \le t \le t_2}\right) < \mathrm{SSB}^{b}_{threshold}|| \tag{19}$$

, where the double bracket || indicates a logical test with outcome 0 (if false) or 1 (if true). Note that the probability is different from the probability that SSB at nearly equilibrium year, $\mathrm{SSB}_{equilibrium}$, falls below the $\mathrm{SSB}_{threshold}$ as follows.

$$\Pr\left[\mathrm{SSB}_{equilibrium} < \mathrm{SSB}_{threshold}|F\right] = \frac{1}{X_b X_s}\sum_{k=1}^{X_b X_s} || \min\left(\mathrm{SSB}^{k}_{equilibrium}\right) < \mathrm{SSB}_{threshold}|| \tag{20}$$

The F-based reference point of F_{ssb} is determined by letting the probability, $Pr\left[\mathrm{SSB}_{future} < \mathrm{SSB}_{threshold}|F\right]$ be equal to a given probability, $Pr_{threshold}$. So, for the calculation of F_{ssb}, $\mathrm{SSB}_t hreshold, Pr_{threshold}$ and projection range of t_1 and t_2 should be specified.

The parameter of $\mathrm{SSB}_{threshold}$ is originally proposed to be $\min(\mathrm{ssb}_{t1 \le t \le t2})$, or average of ten historical lowest (ATHL) of $\mathrm{ssb}_{t1 \le t \le t2}$. Currently it is not determined whether the threshold level should depend on bootstrap iteration (using $\min\left(\mathrm{ssb}^{b}_{t1 \le t \le t2}\right)$), or not (using point estimates of $\min\left(\mathrm{ssb}^{0}_{t1 \le t \le t2}\right)$).

0.6.2 Suites of other reference points

Other reference points based on SPR and YPR are calculated from the following functions. It is noteworthy that the oldest age in estimating SPR and YPR is extended to $3A_{max}$. However, weight at age at the ages older than A_{max} is simply equal to $w_{A_{max}}, w_{A_{max},f}$.

$$
\begin{aligned}
N_a &= \begin{cases} 1 & a = 0 \\ N_{a-i}\exp(-F_{a-i} - M_{a-i}) & a > 0 \end{cases} && (21) \\
C_a &= \frac{F_a}{F_a + M_a}\left(1 - \exp\left(-F_a - M_a\right)\right) N_a && (22) \\
C_{af} &= C_a \mathrm{PFcur}_{a,f} && (23) \\
\mathrm{SPR} &= \sum_{a=0}^{3A_{max}} N_a w_a Q_a && (24) \\
\mathrm{YPR} &= \sum_{a=1}^{3A_{max}} \sum_{f=1}^{nfleet} C_{af} w_{af} && (25)
\end{aligned}
$$

0.7 References

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